What is Space?

by Frank Wilczek

W hat is space: An empty stage, where the physical world of matter acts out its drama; an equal participant, that both provides background and has a life of its own; or the primary reality, of which matter is a secondary manifestation? Views on this question have evolved, and several times changed radically, over the history of science. Today, the third view is triumphant. Where our eyes see nothing our brains, pondering the revelations of sharply tuned experiments, discover the Grid that powers physical reality.

Many loose ends remain in today's best world-models, and some big mysteries. Clearly the last word has not been spoken. But we know a lot, too—enough, I think, to draw some surprising conclusions that go beyond piecemeal facts. They address, and offer some answers to, questions that have traditionally been regarded as belonging to philosophy, or even theology.

A Brief History of Space

Debate about the emptiness of space goes back to the prehistory of modern science, at least to the ancient Greek philosophers. Aristotle wrote that "Nature abhors a vacuum," while his opponents the atomists held, in the words of their poet Lucretius, that

> All nature, then, as self-sustained, consists
> Of twain of things: of bodies and of void
> In which they're set, and where they're moved around.

At the beginning of the seventeenth century, at the dawn of the Scientific Revolution, that great debate resumed. René Descartes proposed to ground the scientific description of the natural world solely on what he called primary qualities: extension (essentially, shape) and motion. Having no properties besides extension and motion, a bit of matter can only be influenced by things that touch it. Thus

to describe, for instance, the motion of planets, Descartes postulated an invisible space-filling "plenum" of invisible matter. He envisaged a complex sea of whirlpools and eddies, upon which the planets surf.

Isaac Newton cut through all those potential complexities by formulating precise, successful mathematical equations for the motion of planets, using his laws of motion and of gravity. Newton's law of gravity doesn't fit into Descartes' framework. It postulates action at a distance, rather than influence by contact. For example, the Sun exerts a gravitational force on Earth, according to Newton's law, even though it is not in contact with Earth. Despite the success of his equations in providing an excellent, detailed account of planetary motion, Newton was not happy with action at a distance. He wrote:

> That one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one another, is to me so great an absurdity that, I believe, no man who has in philosophic matters a competent faculty of thinking could ever fall into it.

Nevertheless, he left his equations to speak for themselves:

> I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called a hypothesis, and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy.

Mathematicians and physicists, through familiarity and spectacular success, became comfortable with the idea of action at a distance through empty space. So things stood, in essence, for more than 150 years.

Over the course of the nineteenth century the plenum struck back. The great experimental physicist Michael Faraday, self-educated and more comfortable with his intuition than with the imposing mathematical machinery of Newtonian physics, envisaged electric and magnetic forces as being transmitted by tubes and lines of force, rather than directly through action at a distance. His highly original viewpoint directed him to experiments that probed these invisible structures— what today we'd call electric and magnetic fields. He thereby discovered simple regularities that would be awkward even to formulate without the fields. The most famous one, which we teach our freshmen, is his law of induction, which says that magnetic fields that change in time create electric fields. (Of course, the actual law is more precise and specific.)

James Clerk Maxwell put Faraday's intuition into precise mathematical form. He introduced electric and magnetic fields that are functions of space and time as the primary ingredients of his theory. He summarized the discoveries of Faraday and others in a system of equations—and found that they were inconsistent! Fortunately, Maxwell realized that he could repair the equations, by adding a new

field based effect. The new effect is a sort of dual to Faraday's induction: Not only do changing magnetic fields create electric fields, but also changing electric fields create magnetic fields.

With Maxwell's addition, fields take on a life of their own; they no longer need particles as their sources. Changing electric fields produce (changing) magnetic fields, which produce changing electric fields, and so forth, in a self-renewing cycle. Maxwell found that his new, improved equations—known today, of course, as Maxwell's equations—have pure-field solutions of this kind, that move along in space at the speed of light. Climaxing a grand synthesis, he concluded that these self-renewing disturbances in electric and magnetic fields *are* light—a conclusion that has stood the test of time.

There is a persistent myth that Einstein, with his special theory of relativity, eliminated the ether. As with most myths, this one is based on a kernel of truth. Einstein indeed discredited some clunky models, popular at the time, which postulated mechanical underpinnings for the electric and magnetic fields. Those crude "ethers" are gone. But the implicit message, that Einstein emptied out space, is dead wrong. Einstein did *not* eliminate space-filling fields as the primary ingredients of physical theory. On the contrary, his special theory of relativity built on the physics of Faraday and Maxwell, and left Maxwell's equations, with its space-filling fields, just as they were.

It is natural to consider entities that fill all space, and have the same properties everywhere, as aspects of space itself.

Cosmic Effervescence

Radio receivers, satellite dishes, or Wi-Fi Internet hookups sense various sorts of excitations in electromagnetic fields; our eyes sense other kinds. Generally, we can track down some source for these signals, *e.g.*, broadcasting stations, the Sun's reflected light, radio and light emission from astronomical bodies. A very special signal, in microwaves, is radiation from far-away hot gas (plasma), finally arriving here after a journey that began just a hundred thousand years after the big bang; this is, of course, the famous microwave background radiation. We can systematically eliminate that and all other sources, say by taking our receiver into a cold, dark, thick-walled cave underground. Having done that, still an ineradicable, faint hum remains. It is random "noise." This irreducible signal represents spontaneous activity of electric and magnetic fields, intrinsic to their nature. It is another, direct indication of the rich inner life of space.

According to the rules of quantum theory, all dynamic entities exhibit spontaneous activity. In particular, fields do. This activity goes by various names: zeropoint motion, quantum fluctuations, or virtual particles. Like most quantum effects this spontaneous activity is barely noticeable in the macroworld, where it is hidden beneath much larger effects from ambient heat (thermal fluctuations). It does, however, become a limiting factor when one designs instruments to detect

extremely weak signals, such as are predicted to arise from gravitational waves or the cosmic axion background.

And in the subatomic realm, spontaneous activity of fields becomes all-important. Spontaneous fluctuations in gluon fields buffet quarks about. Virtual gluons drastically alter the motion of quarks, and causes them, in normal conditions, to bind together into protons and neutrons.

Our eyes were not evolved to resolve the tiny times—10^{-24} seconds—and distances—10^{-14} centimeters—where the action occurs. But we can "look" inside computer calculations, to see what gluon fields are up to. (The calculations have many consequences we can check, so this fantasy is tightly controlled!) To nimbler eyes, space would appear as a self-powered ultrastroboscopicmicronano lava lamp, of which *Figure 1* is a much-magnified snapshot. Creatures with such eyes wouldn't share the human illusion that space is empty.

Not only gluons and quarks, but all forms of matter, in their virtual form, come to be and pass away everywhere and every when. When energy is fed into space, virtual particles become real. They are like magma beneath the surface, ready to erupt if allowed an outlet. In this sense, the cosmic effervescence is the primary reality, from which particles are born.

FIGURE 1

A snapshot of Nothing. Gluon fields are ever fluctuating. Shown here is a typical distribution of energy in the gluon fields. More intense colors correspond to higher energy densities. Densities below a certain cut-off are left transparent, so the structure shows through. (Courtesy of Derek B. Leinweber, University of Adelaide)
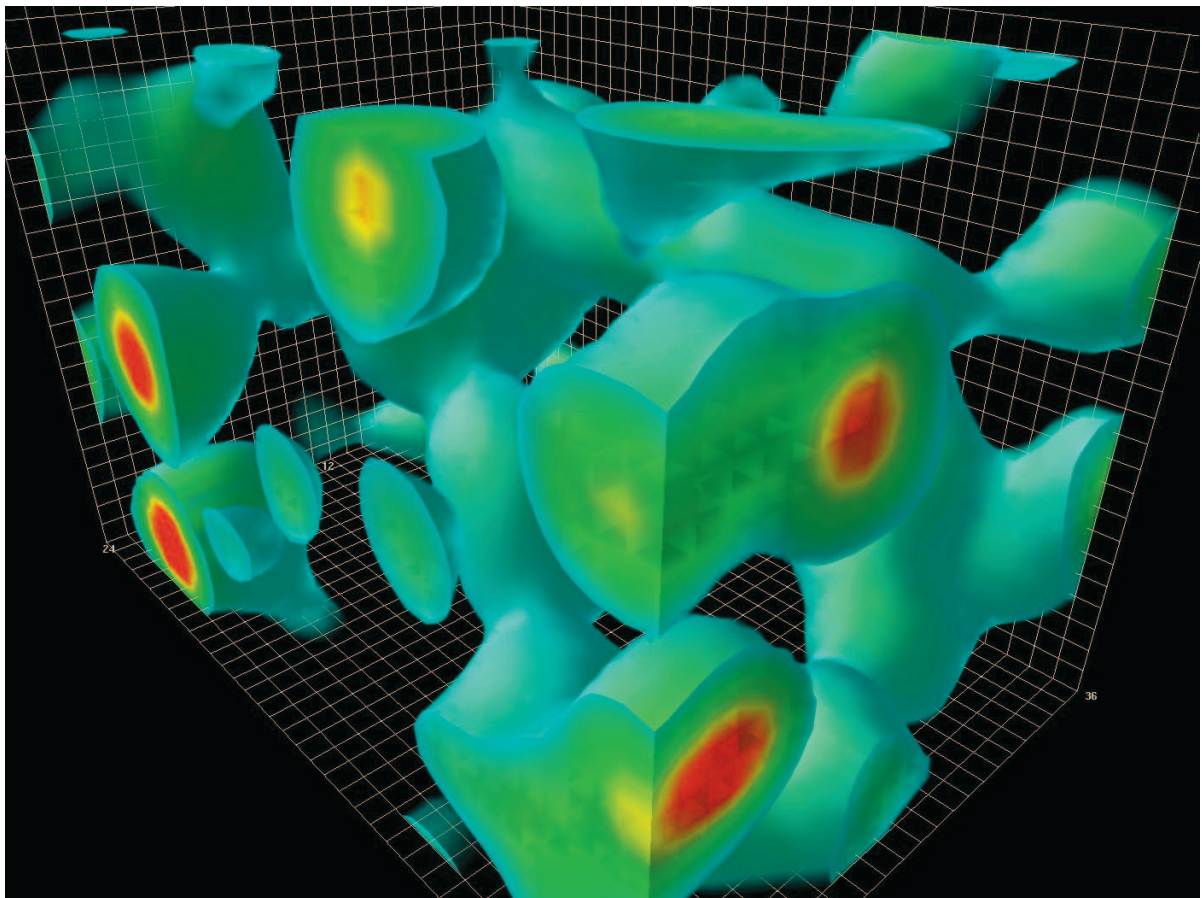
Cosmic Oceans

Besides the fluctuating activity of quantum fields, space is filled with several layers of more permanent, substantial stuff. These are plenums, or ethers, in something closer to the original spirit of Aristotle and Descartes—they are materials that fill space. In some cases, we can identify what they're made of and even produce little samples of it.

Physicists usually call these material ethers *condensates*. One could say that they (the ethers, not the physicists) condense spontaneously out of empty space as the morning dew or an all-enveloping mist might condense out of moist, invisible air.

The best understood of these condensates consists of bound quark-antiquark pairs. Here we are talking about real particles (σ mesons, to be precise), not those ephemeral, virtual particles that come and go spontaneously. The usual name for this space-filling mist of quark-antiquark pairs is "chiral symmetry breaking condensate," but let me just call it what it is: the $Q\bar{Q}$ (pronounced Q-Q bar, for quark-antiquark) background.

The $Q\bar{Q}$ background forms because perfectly empty space is unstable. Suppose that we clean out space by removing the condensate of quark-antiquark pairs. This is something we can do more easily in our minds, with the help of equations and computers, than in laboratory experiments, although as we'll discuss the experiments aren't altogether impossible. Having done this (we compute), quark-antiquark pairs have negative total energy. The mc^2 energy cost of making those particles is more than made up by the energy you liberate by unleashing the attractive forces between them, as they bind into little molecules. So perfectly empty space is an explosive environment, ready to burst forth with real quark-antiquark molecules.

Chemical reactions usually start with some ingredients *A, B* and produce some products *C, D*; then we write

$$A + B \rightarrow C + D$$

and if energy is liberated

$$A + B \rightarrow C + D + energy$$

In that notation, our reaction is

$$\rightarrow Q + \bar{Q} + energy$$

No reactants (other than empty space) required! Fortunately, the explosion is self-limiting. The pairs repel each other, so as their density increases it gets harder to fit new ones in. The total cost for producing a new pair includes an extra fee, for interacting with the pairs that are already there. When there's no longer a net profit, the production stops. We wind up with the space-filling condensate, $Q\bar{Q}$, as the stable endpoint.

An interesting story, I hope you'll agree. How do we know it's right?

One answer is that it's a mathematical consequence of equations—the equations of QCD—that we have many other ways of checking. But while that may be an adequate *logical* answer, since the checks are very detailed and convincing, it's not

exactly science at its best. We'd like our equations to have consequences we can see reflected in the physical world.

A second answer is that we can calculate the consequences having a $Q\bar{Q}$ background, and check whether they match things we see in the physical world. To be more specific, we can calculate whether $Q\bar{Q}$, considered as a material, can vibrate, and what the vibrations should look like. This is very close to what "luminiferous ether" fans once wanted to have for light—a good old-fashioned material, more substantial than electromagnetic fields, whose vibrations describe light. Vibrations of $Q\bar{Q}$ aren't visible light, but they do describe something quite definite and observable, namely π mesons. Among the many strongly interacting particles, the three π mesons have unique properties. They are by far the lightest, for example, and they never fit comfortably within the quark model, which works well for all other strongly interacting particles. So it's very satisfying—and if you study the details, it's very convincing—that they arise in quite a different way, as vibrations of $Q\bar{Q}$.

A third answer is the most direct and dramatic of all, at least in principle. We started by considering the *thought* experiment of cleaning out space. How about doing it for real? Scientists at the relativistic heavy ion collider (RHIC) at Brookhaven National Laboratory, on Long Island, have been working on it, and more such work will be going on at the Large Hadron Collider (LHC). What they do is accelerate two big collections of quarks and gluons moving in opposite directions—in the form of heavy atomic nuclei, like gold or lead nuclei—to very high energy, and make them collide. This is not a good way to study the basic, elementary interactions of quarks and gluons or to look for subtle signs of new physics, because many, many such collisions will be going on at once. What you get, in fact, is a small but extremely hot fireball. Temperatures over 10^{12} degrees (Kelvin, Celsius, or Fahrenheit; at this level, you can take your pick) have been measured. This is a billion times hotter than the surface of the Sun; temperatures this high last occurred only well within the first second of the Big Bang. At such temperatures, the $Q\bar{Q}$ condensate vaporizes—the quark-antiquark molecules from which it's made break apart. So a little volume of space, for a short time, gets cleaned out. Then, as the fireball expands and cools, our pair-forming, energy-liberating reaction kicks in, until the stable $Q\bar{Q}$ background is restored.

All this almost certainly happens. "Almost" comes in, though, because what we actually get to observe is the flotsam and jetsam thrown off as the fireball cools. *Figure 2* is a photograph of what it looks like. Obviously, the photograph doesn't come labeled with circles and arrows telling you what's responsible for what aspect of this spectacularly complicated mess. You have to interpret it. Today, the most accurate and complete interpretations build in the process of $Q\bar{Q}$ melting and re-formation we've been discussing, but they're not yet as clear and convincing as we might hope for. People continue to work at it—both the experiments and the interpretation.

The $Q\bar{Q}$ condensate can be considered an established fact. We vehemently suspect that there are other condensates, too. To explain why, let me start with a parable.
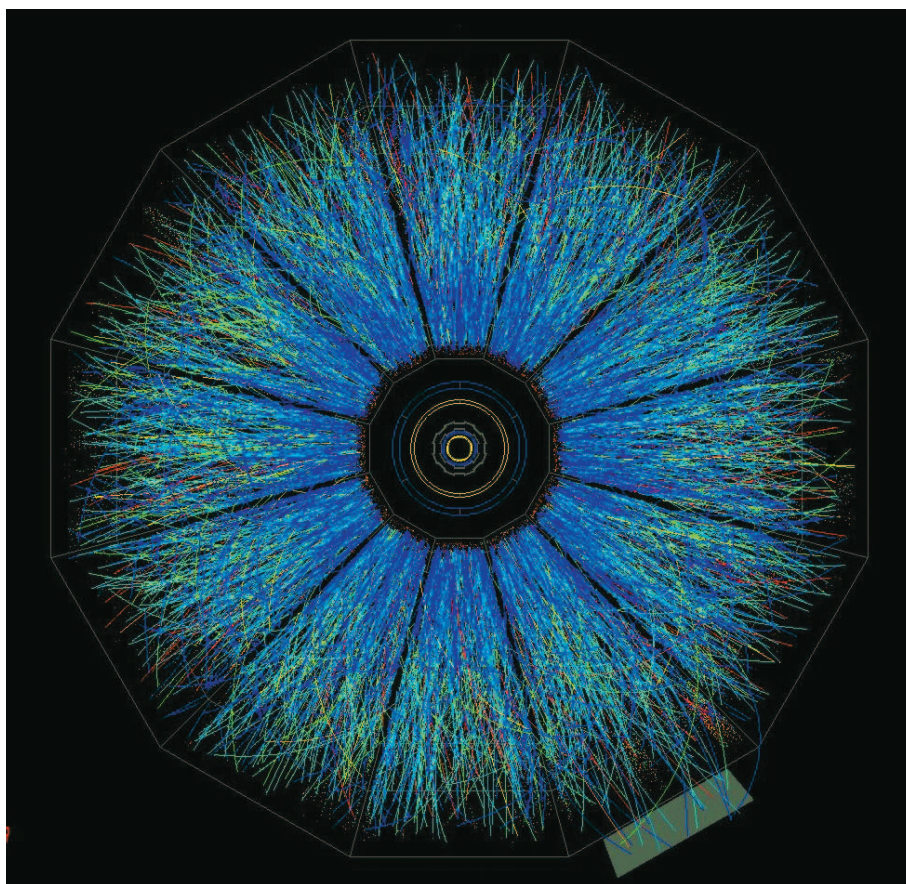
Suppose some species of deepwater fish, that never break the surface, evolved to become more intelligent, and started to do theoretical physics. At first they might formulate the laws of motion for the world they live in, and presumably would take for granted, namely the laws of motion for bodies moving through water. Now we humans know that the laws of motion for bodies moving through water are complicated, and they are not the most basic laws. The laws of motion for bodies moving through empty space are the most basic laws, and you can deduce the more complicated laws for bodies in water by applying those more basic laws to water molecules (deriving hydrodynamics) as well as the bodies moving through them. Eventually the fish-physicists would realize that they could get a nicer version of mechanics by assuming that they lived in a medium—call it water—complicates the appearance of things. In this way, they'd realize that what they hitherto regarded as "nothingness," their ever-present environment, is actually a material medium. And then they might be inspired to do experiments to try to make ripples in the medium, to find its atoms, and so forth.

Well, we're like those fish. Human-physicists have discovered that we can get nicer, simpler accounts of how particles behave by assuming that we're embedded in a medium, whose presence complicates the appearance of things.

The space-filling medium we have the most definite ideas about is called the "Higgs condensate" (after Peter Higgs, a Scots physicist who pioneered some of the ideas). We have a lot of information about how this condensate affects the motion of fundamental particles—especially quarks, electrons and their leptonic brethren, and the *W* and *Z* bosons responsible for the so-called weak interactions. We don't, however, know what it's made out of, or what waves in it do. One idea is that it's made out of just one kind of particle (the Higgs particle), and waves in it just break up into Higgs particles. Other ideas, which I favor, suggest that it's made out of at least two kind of particles, and that waves in it make three additional kinds (similarly to how waves in Q$\overline{\text{Q}}$ make π mesons). It's exciting that the LHC, soon to begin operation at the CERN laboratory near Geneva, should have what it takes, in terms of energy and instrumentation, to produce and detect any of these hypothetical particles.

Elastic Space-Time

In the general theory of relativity, Einstein used the concept of curved space-time to construct a theory of gravity. According to Newton's second law of motion, bodies move in a straight line at constant velocity unless a force acts upon them. The general theory of relativity modifies this law to postulate that bodies follow the straightest possible paths through space-time (so-called geodesics). When space-time is curved, even the straightest possible paths acquire bumps and wiggles, as they must adapt to changes in the local geometry. Putting these ideas together: bodies respond to the topography of space-time. The resulting wiggles in a body's space-time trajectory (in more dignified language, changes in its direction and speed), provide, according to general relativity, an alternative and more accurate description of the effects formerly known as gravity.

We can describe general relativity using either of two mathematically equivalent ideas: curved space-time, or metric field. The metric field is like the legend of a map, which allows a flat chart to represent a bumpy terrain. Mathematicians, mystics, and specialists in general relativity tend to like the geometric view because of its elegance. Physicists trained in the more empirical tradition of high-energy physics and quantum field theory tend to prefer the field view, because it corresponds better to how we (or our computers) do concrete calculations.

Once it's expressed in terms of the metric field, general relativity resembles the field theory of electromagnetism. In electromagnetism, electric and magnetic fields bend the trajectories of electrically charged bodies, or bodies containing electric currents. In general relativity, the metric field bends the trajectories of bodies that have energy and momentum. The other fundamental interactions also resemble electromagnetism. In QCD, the trajectories of bodies carrying color charge are bent by color gluon fields; in the weak interaction, still other types of charge and fields are involved; but in all cases the deep structure of the equations is very similar.

These similarities extend further. Electric charges and currents affect the strength of the electric and magnetic fields nearby—that is, their average strength, ignor-

ing quantum fluctuations. This is the "reaction" of fields corresponding to their "action" on charged bodies. Similarly, the strength of the metric field is affected by all bodies that have energy and momentum (as all known forms of matter do). Thus the presence of a body *A* affects the metric field, which in turn affects the trajectory of another body *B*. This is how general relativity accounts for the phenomenon formerly known as the gravitational force exerted by one body on another. It vindicates Newton's intuitive rejection of action at a distance, even as it dethrones his theory.

Space Weighs and Pushes

Mass has traditionally been regarded as the defining property of matter—the feature that gives substance to substance. So the recent astronomical discovery that space weighs—that the entity we perceive as empty space has a universal, non-zero density—crowns the case for its physical reality.

The concept of space density is essentially the same as Einstein's cosmological term, which is essentially the same as "dark energy." In 1917 Einstein introduced a modification of the equations he originally proposed for general relativity two years earlier. His motivation was cosmology. Einstein thought that the universe had constant density, both in time and (on average) in space, so he wanted to find a solution with those properties. But when he applied his original equations to the universe as a whole, he could not find such a solution. The underlying problem is easy to grasp. Simply put: gravity is a universal attraction, so it is not content to leave things separate. Gravity is always trying to bring things together. So it's not terribly surprising that you can't find a solution where the universe maintains a constant density.

To get the kind of solution he wanted, Einstein changed the equations. But he changed them in a very particular way that doesn't spoil their best feature, namely that they describe gravity in a way consistent with special relativity. There is basically only one way to do that. Einstein called the new term that he added to the equations for gravity the "cosmological term." He didn't really offer a physical interpretation of it, but modern physics supplies a compelling one, which I'll describe momentarily.

The cosmological term can be viewed in two ways. One way, the way Einstein viewed it, is as a modification of the law of gravity. Alternatively, the term can also be viewed as the effect of having a constant density of mass and also a constant pressure everywhere in space and for all time. Since this mass-density and pressure have the same value everywhere, they can be regarded as intrinsic properties of space itself. That's the Grid viewpoint. If we take it as given that space has these properties, and focus exclusively on the gravitational consequences, we arrive back at Einstein's viewpoint.

A key relationship governing the physics of the cosmological term relates its density ρ to the pressure p it exerts, using the speed of light c. It reads

$$\rho = -p/c^2$$

This equation doesn't have a special name, but it will be handy to have one. I'll call it the well-tempered equation, because it specifies the right way to tune the properties of space. Where does it come from?

It's not immediately obvious how a universal, non-zero density can be consistent with special relativity. After all, to an observer moving at constant velocity, objects appear foreshortened in the direction of motion. It would seem, therefore, that the moving observer would see a higher density. But relativity demands that she must see the same physical laws.

The pressure that goes with density, according to the well-tempered equation, provides a loophole. The scales of the moving observer, according to the equations of special relativity, register a new density that is a mixture of the old density and the old pressure—just as, perhaps more familiarly, her clocks register time intervals that are mixtures of the old time intervals and the old space intervals. If—and only if—the old density and old pressure are related in just the way prescribed by the well-tempered equation, then values of the new density (and the new pressure) will be the same as the old values.

Another, closely related consequence of the well-tempered equation is crucial for cosmology. In an expanding universe, the density of any normal kind of matter will go down. But the density of well-tempered grid stays constant! If you're up for a little exercise in freshman physics and algebra, here comes a pretty connection tying that constancy of density directly to Einstein's equivalence of mass and energy. (If not, just skip the next paragraph.)

Consider a volume V of space, filled with mass density ρ. Let the volume expand by ΔV. Ordinarily, as a body expands under pressure it does work, and so loses energy. But the $-$ sign in the well-tempered equation gives us *negative* pressure $\rho = -p/c^2$. So in expanding, our well-tempered grid *gains* energy $\Delta V \times p/c^2$. According to Einstein's mass-energy equivalence, therefore, its mass increases by $\Delta V \times p$. And that's just enough to fill the added volume ΔV with density ρ, allowing space to keep its density constant.

It's possible to measure cosmic density and the pressure separately, using quite different techniques. The density affects the curvature of space, which astronomers can measure by studying the distortion such curvature causes in images of distant galaxies, or—a powerful new technique—in the cosmic microwave background radiation. Using the new technique, by 2001 several groups were able to prove that there was much more mass in the universe than could be accounted for by ordinary matter alone. About 70% of the total mass occurs appears to be very uniformly distributed, both in space and time.

The pressure affects the rate at which the universe is expanding. That rate can be measured by studying distant supernovae. Their brightness tells you how far away they are, while the redshift of their spectral lines tells you how fast they're moving away. Since the speed of light is finite, when we observe the farther-away ones we're looking at their past. So we can use supernovae to reconstruct the history of expansion. In 1998 two powerhouse teams of observers reported that the rate of expansion of the universe is increasing. This was a big surprise, because

ordinary gravitational attraction tends to brake the expansion. Some new effect was showing up. The simplest possibility is a universal negative pressure, which encourages expansion.

The term "dark energy" became a shorthand for both these discoveries—both the additional mass and the accelerating expansion. It was meant to be agnostic about the relative values of density and pressure. If we simply called both of them the cosmological term, we'd be prejudging their relative magnitudes. But apparently we'd be right. The two very different quantities, cosmic mass density and cosmic pressure, observed in very different ways, do seem to be related by $\rho = -p/c^2$.

Thus weighty evidence pushes us to conclude that space weighs and pushes.

Beyond Space

Besides making it more lively, as we've already discussed, quantum theory challenges traditional concepts of space in ways that beggar the speculations of philosophy, mathematics, and science fiction. Of course, after the fact a few philosophers and science fiction writers, and many mathematicians, have built quantum theory into their speculations. But no such free-thinkers anticipated its peculiar weirdnesses, even remotely, before the fact.

The nub of the issue is this. In quantum mechanics, fundamentally, we predict probabilities. Consider, for example, the problem of describing the positions of two quantum-mechanical particles. Quantum theory provides probability distributions $P_1(x)$ for finding particle 1 at position x, if you look for it there, and $P_2(y)$ for finding particle 2 at position y, if you look for it there. We can use three coordinates to define x (or y). So those probability-functions, which are (almost*) the most basic ingredients of the quantum mechanical description, depend on three numbers. They live in ordinary space. But if we ask for the probability of finding particle 1 at x *and* particle 2 at y, when we look for both, then we need a function $P_{12}(x, y)$ that depends on six numbers: three for x, three for y. (This probability is usually not simply the product of the separate probabilities $P_{12}(x, y) \neq P_1(x) \, P_2(y)$. We say the probabilities are "entangled.") So P_{12} does not live in ordinary three-dimensional space, but rather in a six-dimensional configuration space. Similarly, the wave function for positions of three particles lives in a nine-dimensional space, and so forth.

(If you like to stretch your mind, you might enjoy thinking about the probability function for a field $\varphi(x)$. $P(\varphi(x))$ is a function of functions of space; so it lives in a space of infinite dimensions!)

We can boil these considerations down further. When I was a wee lad I liked to put together, and take apart, plastic model rockets. These models couldn't put up satellites, let alone take anyone to the Moon. But they were things I could hold in my hands and play with, and they were aids to imagination. They were built to scale, and there was also a little plastic man on the same scale, so I got a sense of the sizes involved, the difference between an interceptor and a launch vehicle, and

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* Actually the most basic ingredients are wave functions. The value of the probability function is the absolute square of the value of the wave function. This complication is beside the point here.

some key concepts like payloads and detachable stages. Toy models can be fun and useful. Similarly, in trying to understand complicated concepts or equations, it's good to have toy models. A good toy model captures some sense of the real thing but is small enough that we can wrap our minds around it.

In the next few paragraphs I'll show you a toy model of quantum reality. It's a vastly simplified model, but I think it's just intricate enough to suggest the vastness of quantum reality. The main point is that quantum reality is REALLY, REALLY BIG. We'll build up a toy model that describes social life among the spins of just five particles—and discover that it fills out a space of thirty-two dimensions.

Start with one quantum particle that has a minimal unit of spin. We abstract away—that is, ignore—all its other properties. The resulting object is what is called a quantum bit, or qubit. (For sophisticates: A cold electron trapped in a definite spatial state, say by appropriate electric fields, is effectively a qubit.) The spin of a qubit can point in different directions. We'll write

$$|\uparrow\rangle$$

for the state in which the spin of the qubit is definitely up, and

$$|\downarrow\rangle$$

for the state in which the spin is definitely down.

The qubit can also be in states where the spin points sideways, and that's were the fun begins. It's exactly here, at this juncture, that the central weirdness of quantum mechanics comes into play.

The sideways-pointing states are not new, independent states. These sideways-pointers, and all other states of the qubit, are combinations of the states $|\uparrow\rangle$ and $|\downarrow\rangle$ we already have.

Specifically, for instance, the east-spinning state is

$$|\rightarrow\rangle \ = \ \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$$

The state where the spin definitely points east is an equal mixture of north and south. If you measure the spin in the horizontal direction, you'll always find that it points east. But if you measure the spin in the vertical direction, you're equally likely to find that it points north or south. That's the meaning of this strange equation. In more detail, the rule for computing the probability of finding a given result (up or down) when you measure the spin in the vertical direction is that you take the square of the number that multiplies the state with that result. Here, for example, the number $1/\sqrt{2}$ multiplies the spin up state, so the probability of finding spin up is $(1/\sqrt{2})^2 = 1/2$.

This example illustrates, in miniature form, the ingredients that enter into the description of a physical system according to quantum theory. The state of the system is described by its wave function. You've just seen the wave functions for three specific states. The wave function consists of a set of numbers multiplying each possible configuration of the object being described. The number multiplying a

configuration is called the probability amplitude for that configuration. The square of the probability amplitude is the probability for observing that configuration.

More generally, to construct all possible states of two qubits, we add the four possibilities $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$, each multiplied by a separate number. That defines a four-dimensional space—you can step off distances in four different directions.

To describe the possible states of five quibits, we have up-or-down choices for each of them (*e.g.*, $|\uparrow\downarrow\uparrow\uparrow\downarrow\rangle$ or $|\uparrow\uparrow\downarrow\downarrow\uparrow\rangle$). There are $2 \times 2 \times 2 \times 2 \times 2 = 32$ possibilities, and a general state has contributions from every one of them, each multiplied by a number. That's how we find ourselves with a thirty-two dimensional toy model on our hands. Some toy!

Quantum theory forces us to make much more room for physical reality. Traditional concepts of space are paltry by comparison.

Summing Up: Space Today, Space Tomorrow

Space is effervescent, substantial, weighty, and elastic. Each of these properties equates to specific, observable phenomena; they are not whimsical metaphors. Space has a life of its own, and exists independent of any matter that might occupy it. Indeed, in our most fundamental equations particles—the building blocks of matter—are described as disturbances in the activity of space-filling fields, or in other words of space itself.

While they might find details of these views of space surprising, and the concrete evidence for them startling and unfamiliar, Descartes and Newton—or even Aristotle and Lucretius—would have little difficulty understanding what they're about. Philosophers and scientists have argued about similar questions, pro and con, for centuries. The progress is that now we have some answers.

Modern quantum physics brings in ideas of a different order. Quantum reality lives in spaces whose meaning, size, and structure transcends classical ideas about physical space. To get in tune with Nature, we must vastly expand our conceptual universe.

With new answers come new questions. The structure of space is encoded in the metric field. Like all fields the metric field is subject the laws of quantum mechanics. In particular, it is forever boiling with spontaneous fluctuations. When we calculate these fluctuations we find that they grow, as a fraction of the distance, for nearby points. Eventually, for distances below about 10^{-33} cm., the calculated fluctuations in distance become larger than the distance itself. Below this so-called "Planck length" our usual methods of calculation break down. Indeed, the whole concept of distance comes to look suspect. Now 10^{-33} cm. is a *very* small distance, far beyond practical access. Nevertheless this issue is of fundamental interest, not only in its own right, but also for cosmology. Indeed, our equations break down in describing extremely short time intervals (10^{-44} sec.), for similar reasons. Thus we aren't able to describe the very earliest moments of the big bang. And so ultimate questions of origins remain up for grabs.

Many centuries ago, Saint Augustine was troubled by the question "What was God doing before He created the world?" In a brilliant (but pre-scientific) analysis, Augustine examined the concept of time, and proposed—roughly speaking—that without finite minds to apprehend it, or clocks to measure it, time as we know it ceases to exist. Thus, according to Augustine, time itself came into existence together with the created world—not before! And so, his troublesome theological question evaporated. Maybe a similar fate awaits our modern physical analogue: "What happened before the big bang?" Or maybe not.

In any case, we've learned for sure that there's much more to the world than meets the eye. Seeing is an active process. The harder you look, the more you find. Where our unaided eyes see Nothing, our creative minds, through vigorous investigation, discover very much indeed.

PROFESSOR FRANK WILCZEK *is considered one of the world's most eminent theoretical physicists. He is known, among other things, for the discovery of asymptotic freedom, the development of quantum chromodynamics, the invention of axions, and the discovery and exploitation of new forms of quantum statistics (anyons). When only 21 years old and a graduate student at Princeton University, in work with David Gross he defined the properties of color gluons, which hold atomic nuclei together.*

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