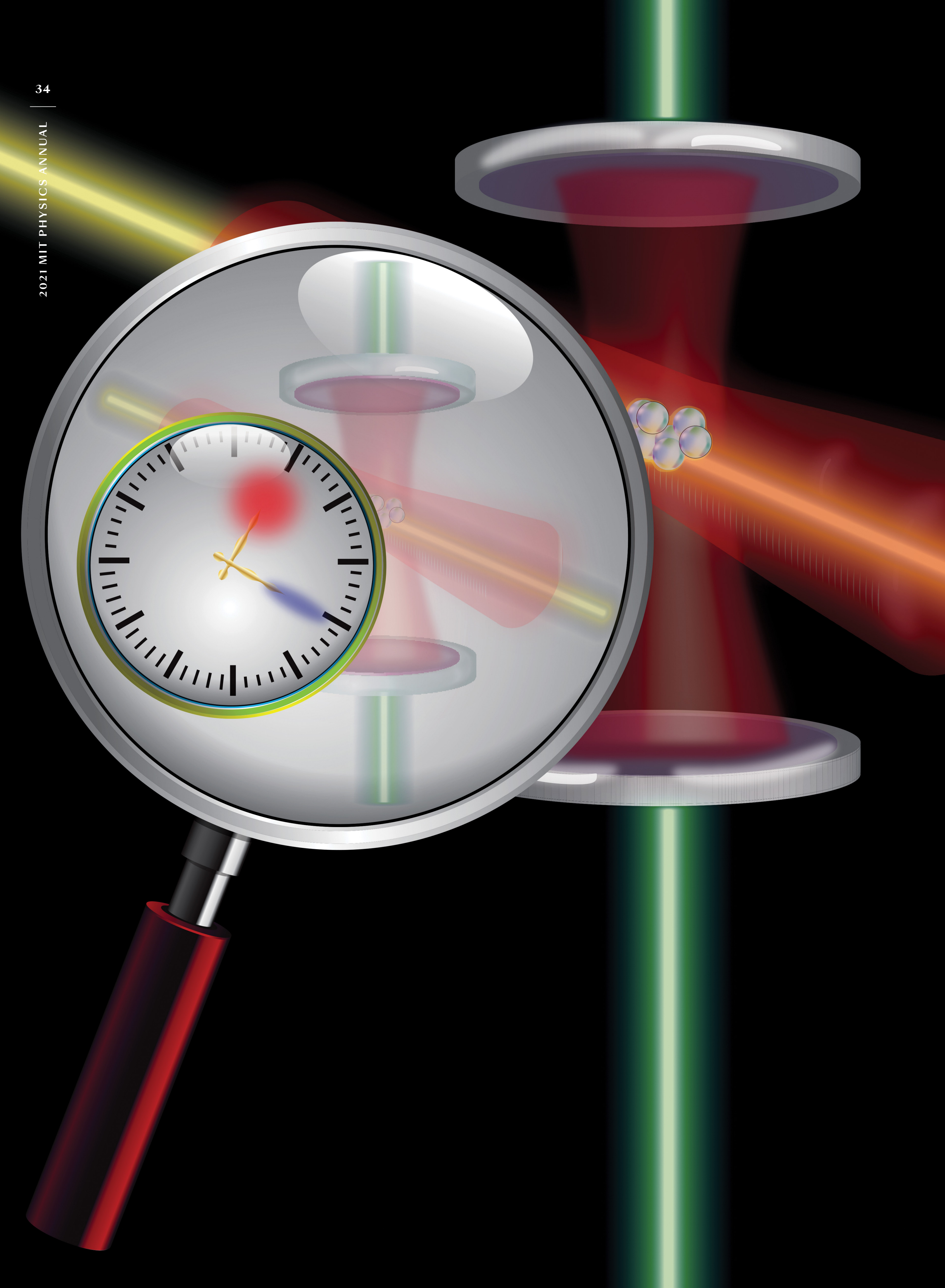


Keeping Better Time through Entanglement

by Vladan Vuletić, Simone Colombo and Edwin Pedrozo-Peñafiel

How to accurately measure time? Since time immemorial, humans have tried to keep track of the passing of time. How can one measure time? In a somewhat circular definition, time is measured through a periodic or repetitive process where each period takes a constant time.

For millennia, the best time standard that humanity had was the motion of celestial bodies, as evidenced by the length of the day, the lunar calendar or the solar year. In fact, until 1960 the standard of time was based on the Earth's motion around the Sun, *i.e.*, the astronomical year. However, by the middle of the last century it had been experimentally found that certain internal oscillations in atoms can be more stable than the motion of the Earth around the Sun, which is being influenced by the constellation of other planets. Hence, for the last 60 years time has been defined through the oscillation of a cesium atom between the hyperfine levels of its electronic ground state, with exactly 9,192,631,770 oscillations constituting 1 second.

Figure 1 shows the progress in accuracy made by cesium clocks over the past decades (blue data points). The improvement of cesium clocks follows an exponential Moore's law with about one order of magnitude in precision gain every decade. After 2000, a new type of clock was introduced, enabled by breakthroughs in laser and atom cooling and trapping technologies. These new devices are optical clocks that measure not the oscillations between hyperfine states at microwave frequencies (in the 10^{10} Hz range) like the cesium clock, but instead keep track of oscillations at the 10^5 times higher optical transition frequencies of 10^{15} Hz. Due to their much higher oscillation frequency, such clocks quickly started

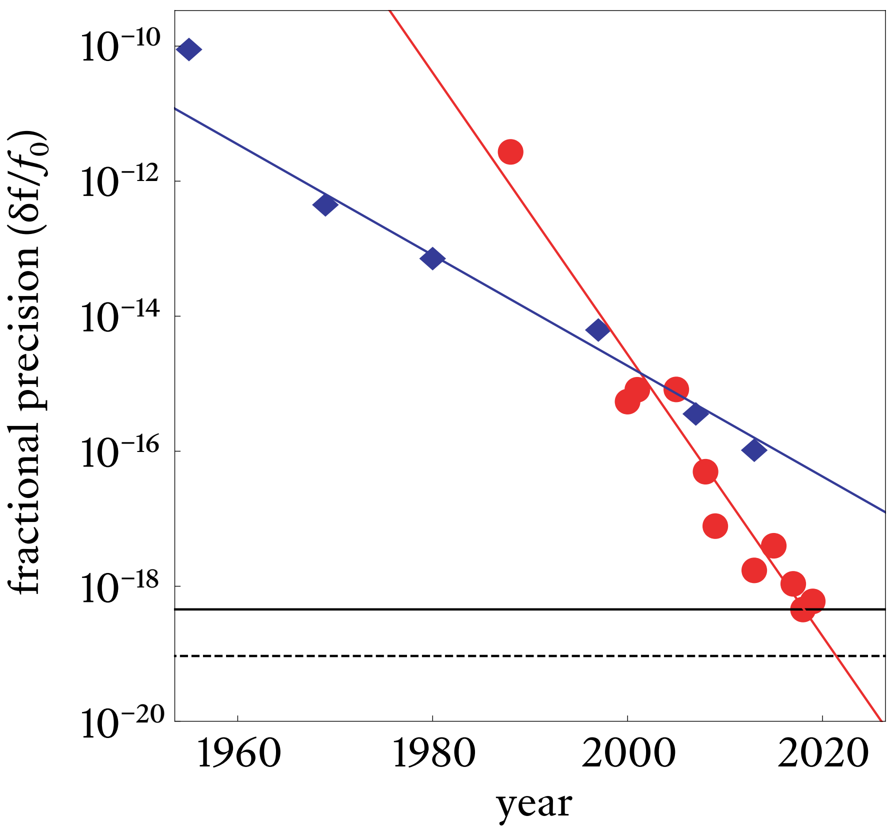


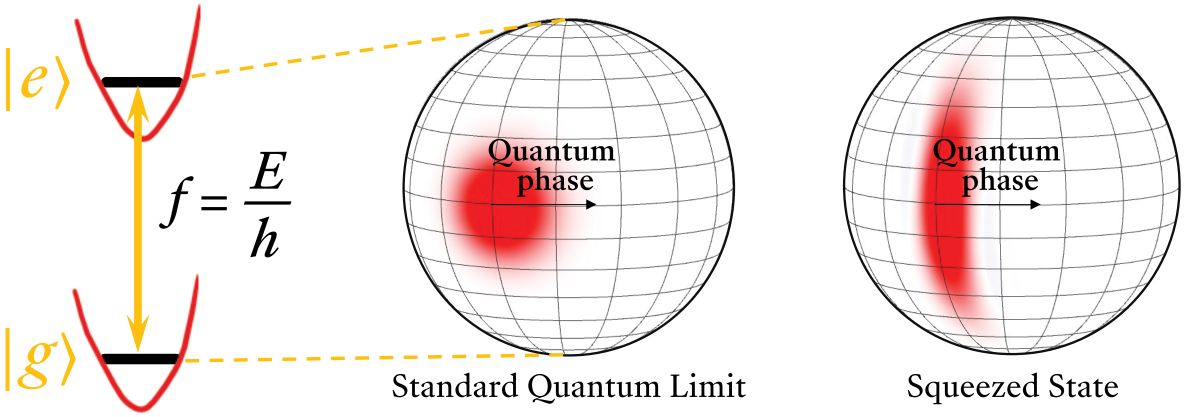
FIGURE 1:
Progress in atomic clock precision over the last decades. The blue data points represent microwave clocks using cesium atoms, while the red data points represent clocks that operate on an optical transition. The solid black line represents the Standard Quantum Limit of an ytterbium optical-transition clock operated with five seconds of repeated interrogation time for one hour using 1,000 atoms. The Standard Quantum Limit is the best performance that can be achieved with independent, *i.e.*, not entangled, atoms. Credit: Vuletić Group

outperforming microwave clocks (red data points in *Fig. 1*). Today, optical-transition clocks are approaching a mindboggling fractional stability below 10^{-19}, equivalent to an error of only a few milliseconds for a hypothetical clock running since the Big Bang. This precision also means that one needs to be very careful about controlling the gravitational environment when measuring time: According to Einstein's general theory of relativity, time passes more slowly in a more negative gravitational potential, and current clocks are already sensitive to centimeter-scale differences in the clock height in the Earth's gravitational field.

How do atomic clocks achieve such exquisite performance? The working principle of atomic clocks is the conversion of an energy difference E between two atomic levels into a frequency f or oscillation period $T = 1/f$ via Planck's quantum h, $f = E/h$. To achieve high precision, atomic clocks use a long-lived electronically excited state with a typical lifetime τ of 10 seconds or more, corresponding to a high quality factor of the oscillation $Q = f\tau > 10^{16}$. Furthermore, as described by the Fourier theorem, high frequency or time resolution can only be achieved if the interrogation time is long: a precision of 1 Hz in one atom requires an interrogation time of one second. Atoms exhibiting random motion at room temperature would be leaving the interrogation laser beam far too quickly, but by laser cooling them and holding them in a trap, long clock interrogation times up to several seconds can be achieved. The trap, however, needs to have quite special properties, since it must affect exactly equally the energies of the two atomic states so that the energy difference E is not changed by the trap. This is accomplished by using a laser beam trap that is tuned to a 'magic wavelength' such that the optical polarizabilities for the ground state and the electronic excited state are exactly the same (*Fig. 2*). Finally, most optical-transition atomic clocks use many atoms, typically between 10^3 and 10^5, to improve the signal-to-noise ratio.

Quantum limitations of atomic clocks and entanglement

Atomic clocks operate by creating a quantum mechanical superposition of the ground state $|g\rangle$ and an excited state $|e\rangle$, and measuring the evolution frequency $f = E/h$ of the quantum mechanical phase between the two levels. The two-level system consisting of $|g\rangle$ and $|e\rangle$ can be represented formally as a (pseudo-) spin-½ system. A collection of N identical atoms constituting a clock can then be considered as a large spin $S = N/2$ that can be visualized as a vector $\vec{S}$ on a sphere of radius $\sqrt{S(S+1)}$, the so-called Bloch sphere (*Fig. 2*). The clock oscillation of the quantum phase $|g\rangle + e^{-iEt/h}|e\rangle$ then corresponds to a rotation of the effective spin vector $\vec{S}$ around the equator of the Bloch sphere. To measure this rotation, the accumulated phase, or equivalently, the direction of the angular-momentum vector in the equatorial plane, is compared to the oscillation phase of a laser that operates at very nearly the same frequency. For a probing time of 1 second, $\vec{S}$ has performed as many as 10^{15} rotations, and the electric field of the laser beam has also oscillated the same number of times. The atomic phase, or equivalently, the direction of $\vec{S}$ in the equatorial plane, is then used as feedback to stabilize the laser phase.

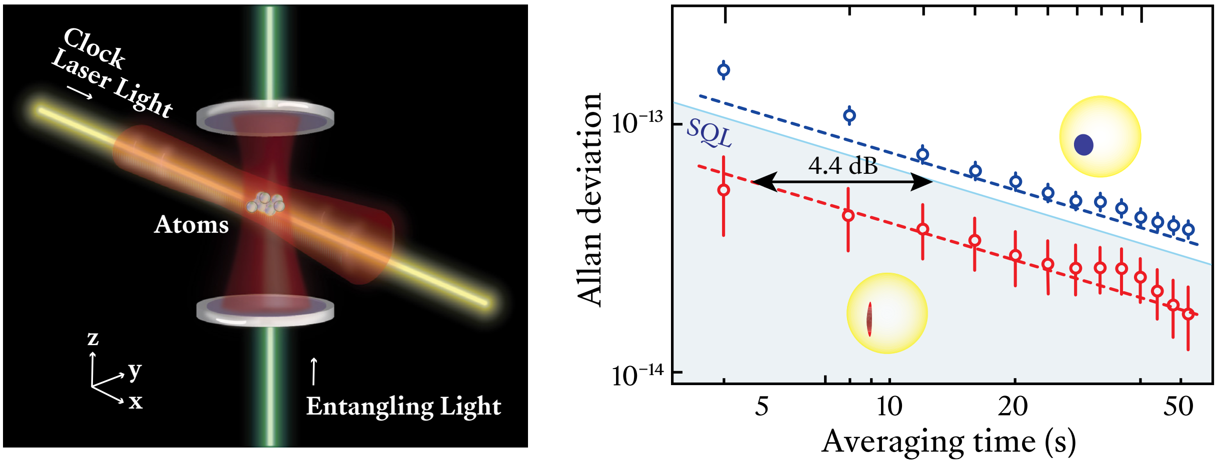


While the direction of a classical angular momentum can in principle be measured perfectly, the quantum mechanical spin $\vec{S}$ is subject to Heisenberg uncertainty rules, which impose a non-zero uncertainty in its direction, or equivalently, a fundamental uncertainty in the measurement of the quantum mechanical phase (*Fig. 2*). This can be traced to the discrete nature of measurements in quantum mechanics. Measuring a component of a spin ½ in a superposition state $|g\rangle + |e\rangle$ is like a coin toss where only after many repetitions of the toss it is possible to access the probabilities for head or tail with some accuracy. The corresponding binomial distribution for N coin tosses (or measurements on N spin-½ particles) gives rise to the so-called Standard Quantum Limit where the precision of the experiment improves with the number of coin tosses (or particles) as $N^{-1/2}$. This is a fundamental limit on measurements with independent particles that exists even after all noise of a technical nature has been eliminated. State-of-the-art optical clocks typically operate near the Standard Quantum Limit.

FIGURE 2:
Two-dimensional magic wavelength optical-lattice trap holding ytterbium atoms at micro-Kelvin temperature for the realization of the entanglement-enhanced optical atomic clock. (p. 38) The two atomic levels are $|g\rangle$ and $|e\rangle$, and the N two-level systems are represented on the generalized Bloch sphere as an effective total spin $\vec{S}$. The top-middle and top-right distributions on the Bloch spheres represent an unentangled state of independent atoms and a squeezed spin state, respectively. The projection noise of the final measurement, or equivalently, the Heisenberg uncertainty rules for angular momentum, impose an uncertainty in the direction of the total spin $\vec{S}$. The squeezed spin state using entangled atoms has a lower quantum noise in the phase direction, *i.e.*, enables better frequency resolution. (p. 39, on left) Experimental setup. (page 39, on right, adapted from [7]) Clock uncertainty (Allan variance) vs. averaging time, comparing a clock using as input states an unentangled state (blue) and a squeezed spin state (red), respectively. The entangled state outperforms the Standard Quantum Limit by 4.4 dB. Credit: Vuletić Group

However, the Standard Quantum Limit is not an absolute limit in a many-particle system if one allows for the possibility to establish quantum correlations (entanglement) between the particles. The simplest states with metrologically useful entanglement are so-called squeezed spin states [1], where the quantum noise of the spin is reduced (squeezed) in one quadrature at the expense of another quadrature that is not directly relevant to the measurement. In particular, as shown in Figure 2, one can redistribute the quantum noise from the phase quadrature (that measures time) into the S_z quadrature that to lowest order does not affect the measurement of phase or time.

Squeezed spin states had been proposed in the 1990s as a possibility to improve over the Standard Quantum Limit [1,2], but it was not until twenty years later that it became possible to entangle the quantum states



of many atoms with each other. The breakthrough was provided by laser and optical-resonator technology, when we realized that light circulating inside a resonator can be used as a messenger between atoms that can induce many-body quantum correlations in the atomic system [3]. In 2010, a group led by Eugene Polzik in Copenhagen [4] and our group at MIT [3,5] independently demonstrated the first atomic spin squeezing using light. (A method using atomic collisions in a Bose-Einstein condensate had been demonstrated a little earlier by a group in Heidelberg led by Markus Oberthaler [6], but that method is not suitable for precision experiments due to large uncontrolled collision-induced clock shifts.) We were also able to show in 2010 that a microwave clock could indeed be improved by spin squeezing, when we achieved an improvement by a factor of three over the Standard Quantum Limit.

However, those results were mainly first proof-of-principle experiments with microwave clocks that were operating far from the precision that can be achieved with state-of-the-art clocks. We then proceeded to build an apparatus that enables spin squeezing and performance beyond the Standard Quantum Limit in an optical-transition clock using ^{171}Yb atoms, one of the two frontrunner clock types in the field. In 2020 we demonstrated for the first time that an optical-transition clock can be spin squeezed and operated beyond the Standard Quantum Limit (*Fig. 2*) [7]. The step from microwave to optical clocks took almost a decade in part because, due to the 10^5 times larger energy difference between atomic levels and associated phase evolution rate $E/\hbar$, it is much more difficult to maintain entanglement in the optical domain than in the microwave domain.

Reversing time

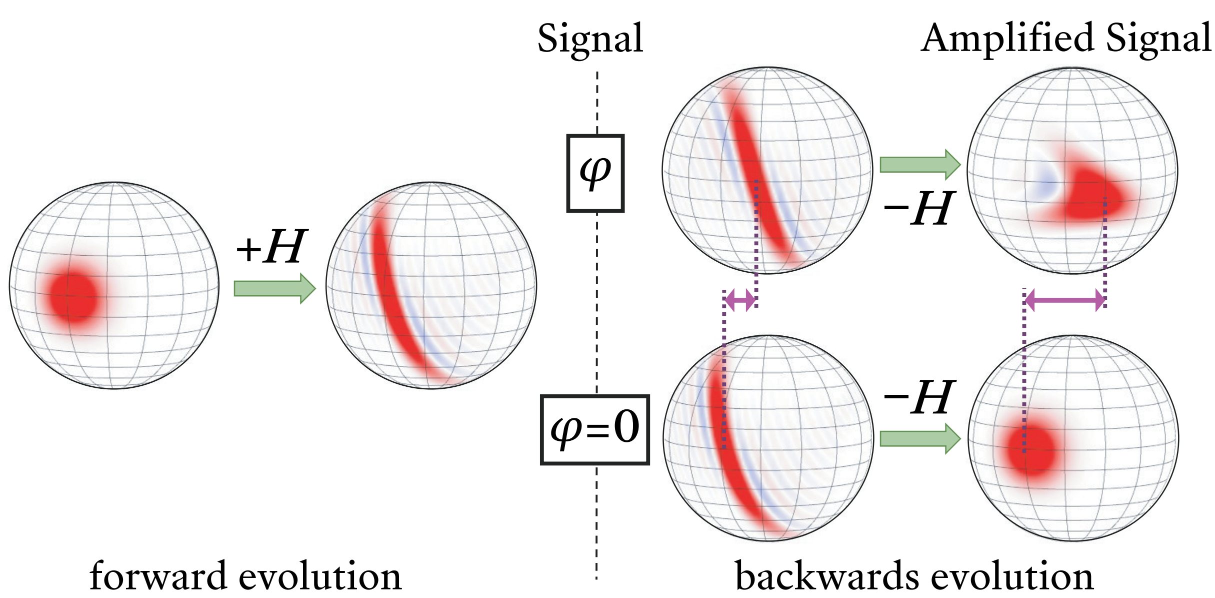
It is possible to create more complex many-body entangled states than the squeezed spin state, and such states can also potentially offer even more improvement over the Standard Quantum Limit. A particularly interesting possibility is the generation of an evolution effectively backwards in time by switching the sign of a many-body Hamiltonian H. Since the evolution of a quantum state is governed by the operator $U = exp(-iHt/\hbar)$, such a sign change from H to $-H$ is equivalent to an evolution backward in time under the original Hamiltonian H.

It turns out that this type of time-reversal process can be used for quantum metrology well beyond the Standard Quantum Limit. Furthermore, this process may potentially allow one to operate a clock close to the truly fundamental Heisenberg Limit. The latter is determined by the Heisenberg uncertainty rules for angular momentum, and sets a limit N^{-1} to the improvement of the clock precision with atom number N, as opposed to the $N^{-1/2}$ Standard Quantum Limit. For 10^4 atoms, a clock at the Heisenberg limit could outperform the Standard Quantum Limit by a factor of 100.

We have recently demonstrated such a scheme where a strongly entangled state is generated by a many-body Hamiltonian (*Fig. 3*). This state is highly sensitive to small displacements, and if such a displacement occurs, it can be made directly visible after an evolution "backwards in time" with the negative Hamiltonian. This effectively leads to an entanglement-induced signal amplification that enables operation of a quantum sensor or clock well beyond the Standard Quantum Limit, and at fixed distance from the Heisenberg Limit as we vary the atom number (*Fig. 4*). We achieve a precision improvement that is linear in the atom number, rather than improving only as the square root of the atom number. This system also yields the highest gain, by a factor of 15, beyond the Standard Quantum Limit that has been demonstrated by any interferometric device demonstrated so far.

Over the last two decades, many-body entanglement has developed from a pure basic research area to a useful tool to improve atomic clocks and other quantum sensors. Interestingly, precision clock that use entanglement have much in common with quantum simulators and quantum computers: the need to preserve quantum mechanical superposition states for long times, to perform controlled

FIGURE 3:
Quantum metrology with many-body entangled states based on time reversal. An entangled state with a strongly non-Gaussian envelope is generated by the action of a many-body Hamiltonian (+*H*). This state is then first subjected to a small displacement, and then to a negative Hamiltonian (-*H*), which generates an evolution effectively backwards in time. This results in a strong amplification of the small signal. The time-reversal protocol enables the use of highly entangled states for quantum metrology while performing a simple final measurement, removing the need for high measurement resolution. Credit: Vuletić Group



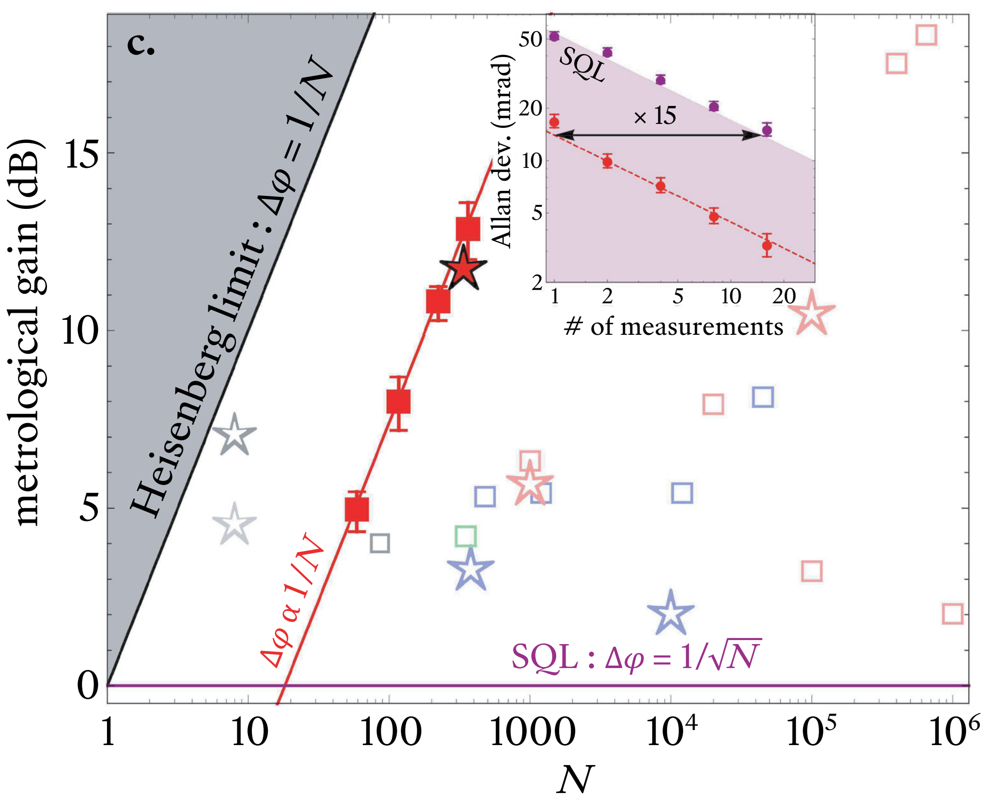


FIGURE 4:
Heisenberg scaling of sensitivity with atom number. The Heisenberg limit for phase detection, scaling with atom number as N^{-1} is shown. Filled red squares represents our experimental data showing N^{-1} Heisenberg scaling in precision, and being 12.6 dB away from the Heisenberg limit. For comparison, results from previous experiments using Bose-Einstein condensates (blue empty squares), thermal atoms (red squares), ions (black squares), and Rydberg atoms in tweezer arrays (grey squares) are shown. The stars correspond to phase measurements in a full interferometric sequence following the same color code. Our measurement (filled red star) shows the best phase sensitive beyond the Standard Quantum Limit, (11.8 ± 0.8) dB. The inset shows the clock instability (Allan variance) for phase measurement where we observe an improvement by a factor of 15 in sensitivity when compared to the Allan variance of an unentangled state. Credit: Vuletić Group

state transformations, and to utilize entanglement to achieve system properties that cannot be attained by classical systems. Compared to quantum simulators, entangled atomic clocks typically use less complicated many-body quantum states but (many) more atoms. In the past, there has been significant cross-fertilization between the fields, with ideas from quantum information science strongly influencing the development of quantum metrology.

Finally, as clocks start breaking the 10^{-20} barrier of fractional stability, gravitational effects on time need to be seriously considered: A difference in clock height of 1 mm corresponds to a gravitational red shift of 10^{-19}. Thus to compare two clocks at the 10^{-20} level, one needs to establish their relative height difference in the Earth's gravitational potential to better than 100 μm. How does one compare clocks across the United States, let alone across continents, at this level? On the other hand, with such a high precision, there may be new fundamental effects influencing the passing of time awaiting to be discovered. These include new unknown physics such as the possibility that our fundamental constants, *e.g.*, the speed of light or the fine structure constant, are changing as the Universe is expanding. Atomic clocks and quantum entanglement may thus, through a new precision window, open a glimpse into the inner workings of our world.

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