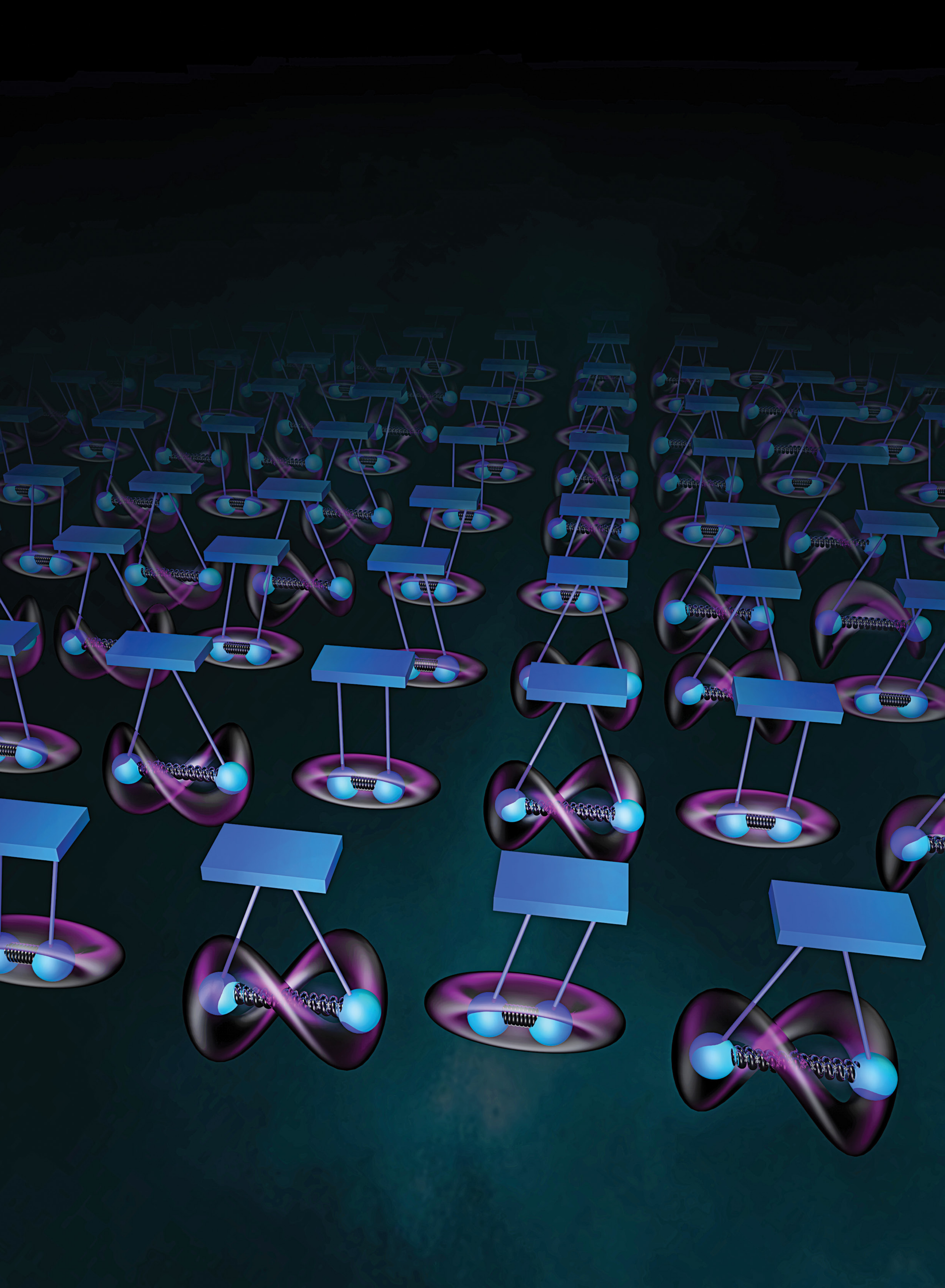


The Quantum Pendulum Qubit

How atomic vibrations can be brought into long-lasting quantum superpositions

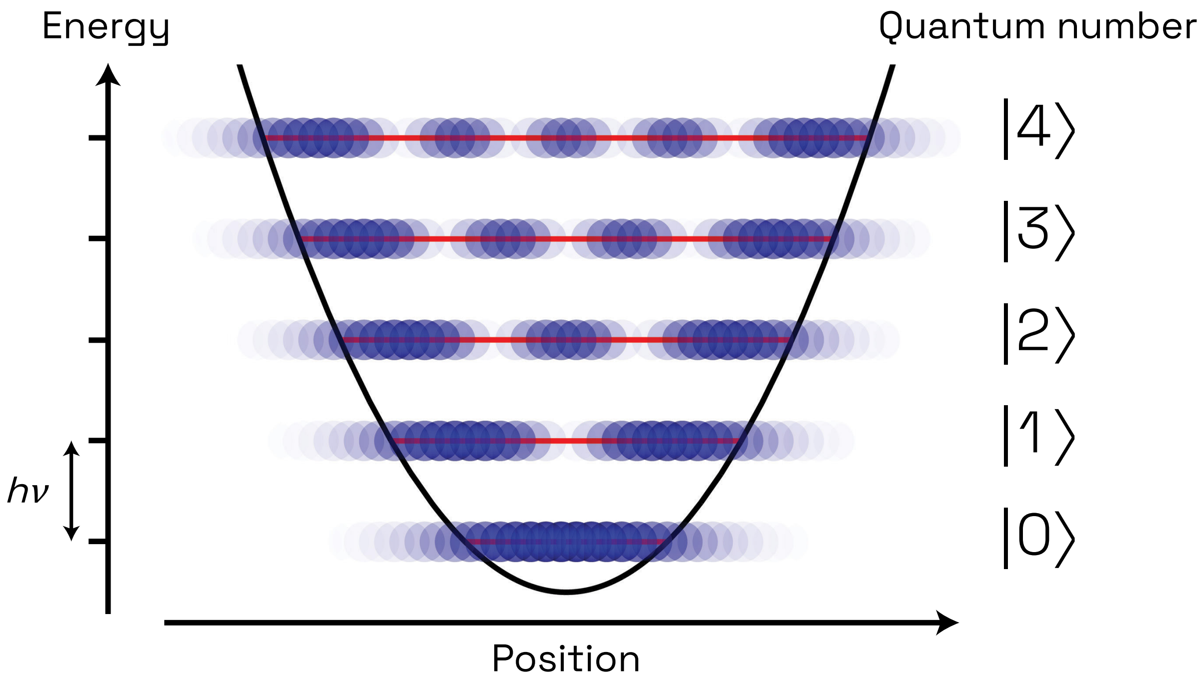
by Martin Zwierlein

The harmonious swing of a pendulum has fascinated humankind for centuries, from children to physicists. Its thorough analysis by Galileo and Heisenberg has given us classical and quantum mechanics, and by analogy revealed the nature of light and matter. In the quantum world, a pendulum bob may exist in a quantum superposition of being here and there, of swinging to and fro. In a recent experiment at MIT, an atomic version of a double pendulum displayed superpositions of vibrational states that lasted ten seconds—a promising foundation for quantum computers.

Galileo took his pulse. Indeed, the chandelier up in the cathedral's ceiling swung back and forth with remarkable constancy, seemingly independent of how strong the wind had pushed it. This moment, some 440 years ago in Pisa, marks the birth of the pendulum clock and quite well the beginning of modern physics. Galileo rushed home, testing pendulum motion with a variety of masses and string lengths, and found that a pendulum's period only depended on the length of the string, and not on the amplitude of the swing. In passing, he hereby invented the experimental method, to test nature rather than to philosophize about its workings.

Ever since Galileo, his idea of measuring time through vibrating motion has been perfected to an astounding degree. Christian Huygens invented the first practical pendulum clock and showed that only for small excursions the period is independent of the swing's amplitude; we say the motion is slightly anharmonic. The precision of his clocks was remarkable, showing an error of only tens of seconds in one day. Today, we still measure time using vibrations, but it is now the vibrations of electrons in atoms, and we would err by less than a second in the entire age of the Universe.

Between then and now, one revolution has completely changed the way we physicists think about motion: quantum mechanics. To understand spectra of atomic gases, Niels Bohr postulated that electrons can only move along particular orbits around the nucleus, an incredibly daring and successful assumption, but the reasons for its success remained obscure. In 1925, Werner Heisenberg, on retreat in Helgoland to alleviate his hay fever, carefully contemplated the simpler problem of the pendulum [1]. He suddenly saw clearly that one had to give up the entire notion of precisely determined paths that the pendulum bob follows. Indeed, if we did precisely know the bob's location at some point in time, that implied its velocity to be completely uncertain, and the next moment it would be anywhere. The delicate balance between position and momentum uncertainty is only stably maintained for certain values of the pendulum's energy (*Fig. 1*). There is a lowest vibrational state, which we may call $|0\rangle$, which has the pendulum bob not actually at rest (again, then we would precisely know where it is), but in a state of minimum uncertainty in both position and momentum. The first excited state, $|1\rangle$, of the pendulum, has higher energy than the ground state by an amount $h\nu$, Planck's constant h times the frequency of the pendulum ν. This is, not accidentally, Einstein's relation for the energy of a photon. An entire ladder of vibrational states $|2\rangle$, $|3\rangle$, ...is built up from there, each approximately $h\nu$ higher in energy than their predecessor.



For Galileo's pendulum, this energy is tiny, about 10^{-34} calories, and therefore irrelevant. But this "energy quantization" of oscillators is readily seen with atoms, electrons, and, at low temperatures, even with entire mechanical oscillators. Interestingly, if the pendulum is in any of these states, nothing is actually vibrating. If I repeatedly measure the location of the bob, then averaged over many measurements, I will find it hanging straight down. To see its average position swing back and forth, the pendulum needs to be in a superposition of two energy states, so at once in $|0\rangle$ as well as in $|1\rangle$, for example. The ability in quantum mechanics to create superpositions of quantum states is most amusingly illustrated by Schrödinger's cat, which is placed in a superposition of |dead⟩ and |alive⟩ until someone comes and checks on it.

FIGURE 1:
The Quantum Pendulum. An oscillator such as the pendulum features a (nearly) parabolic potential landscape (black curve). In quantum mechanics, the position and velocity of the pendulum bob are uncertain. We can only give a probability to find the bob at a certain location, indicated by the opacity of the bobs above. Only discrete values of the bob's energy are allowed (red lines), the quantum states are labelled $|n\rangle$ with an integer n, and spaced (approximately) by $h\nu$, Planck's constant times the pendulum's frequency.

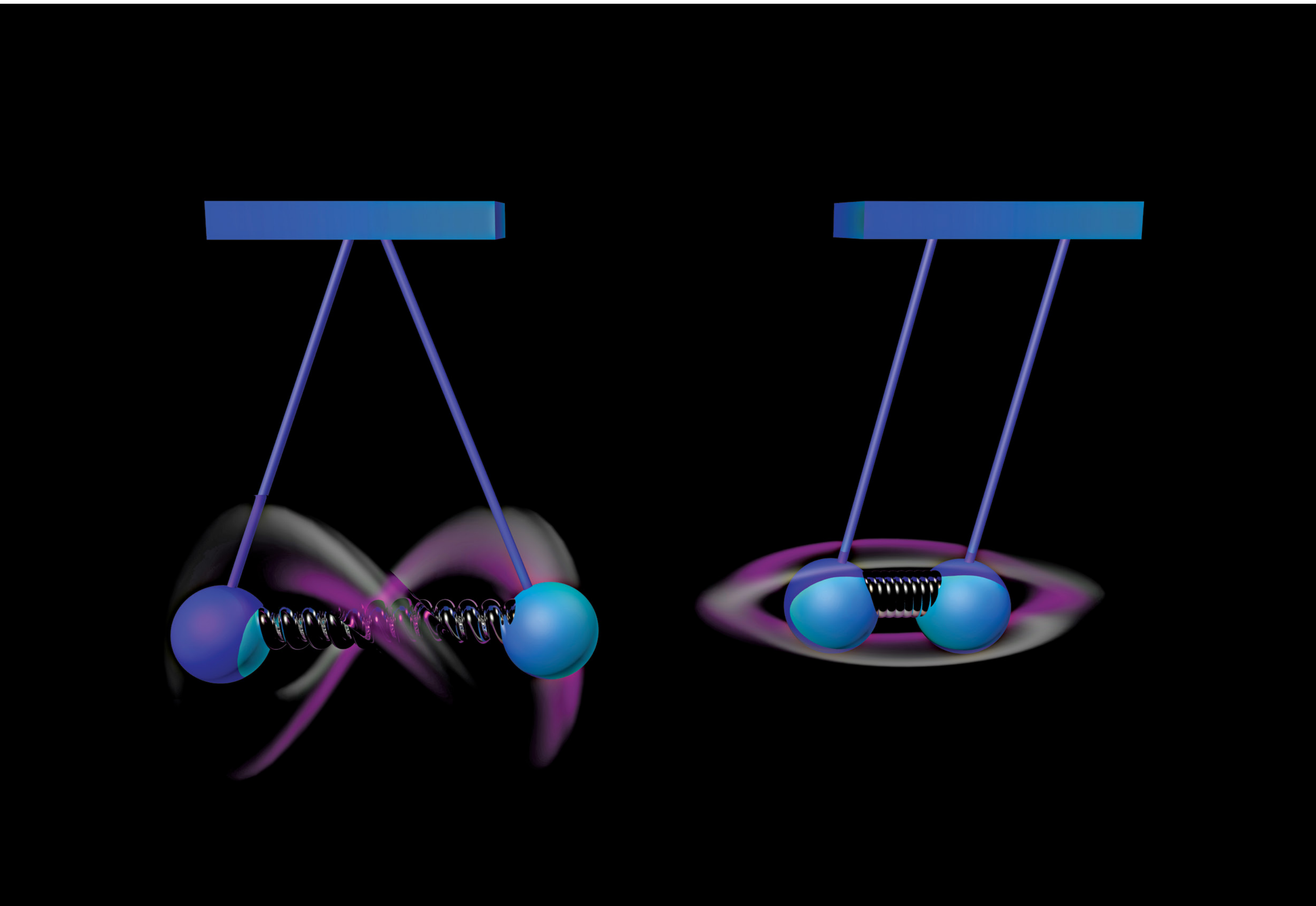
A second quantum revolution

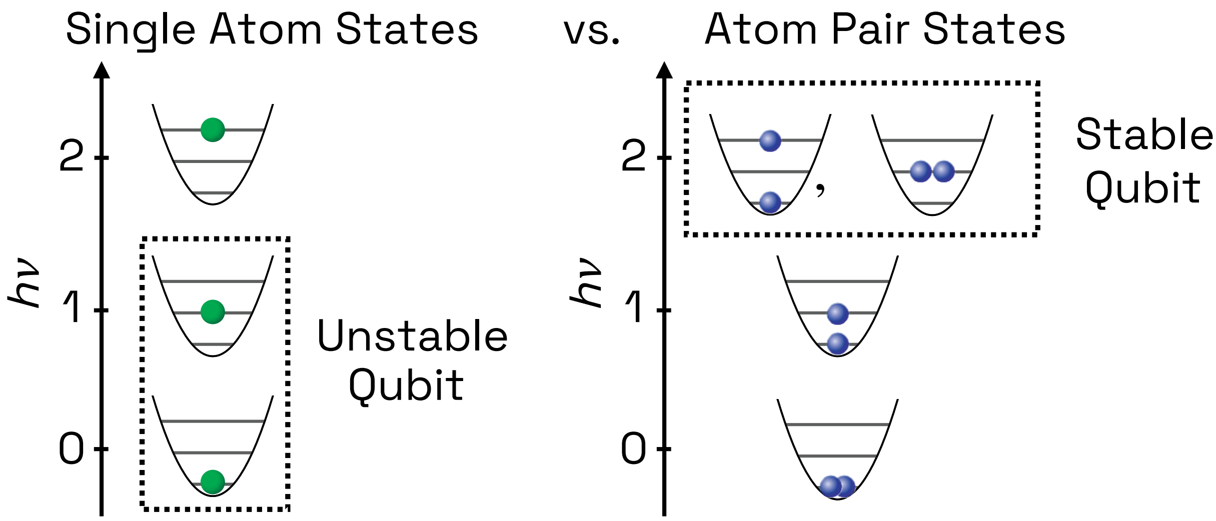
We are currently witnessing a second "quantum revolution," started by MIT's Peter Shor realizing in 1994 that quantum superpositions, like of $|0\rangle$ and $|1\rangle$ above, allow factorizing numbers exponentially faster than on a classical computer. Instead of working serially with bits like 0 and 1, a quantum computer works with qubits, which can store such superpositions of $|0\rangle$ and $|1\rangle$. Promising qubit architectures for quantum computers are electronic states in neutral atoms, in ions, and in superconducting circuits. One difficulty for all platforms is to maintain the quantum superposition for a long time, lasting long enough for the algorithm to complete. Another concern is how to increase the number of qubits to actually perform calculations that cannot be done with classical computers.

Could we actually use a quantum version of Galileo's pendulum to store qubits? Neutral atoms can be trapped in the focus of a laser beam, and the restoring force of the light acts like gravity on Galileo's pendulum. What is more, by interfering laser light we can create entire "crystals of light," periodic arrays that can trap thousands of atoms, one in each well. And thanks to the techniques of laser cooling and evaporative cooling (which brought us Bose-Einstein condensates, the coldest matter of the Universe; see Wolfgang Ketterle's articles in *physics@mit* 1997 and 2001), atoms can be brought to all occupy the ground state $|0\rangle$ of their trap. It is tempting to think that such an array of "atomic pendula" is a good starting point to build a quantum register, a collection of qubits that can store quantum information.

However, here we come back to Galileo's discovery that the frequency of the pendulum depends on its length. In the laser trap, the role of the length is played by a combination of laser power and how tightly we focus it. It turns out to be difficult to have all our pendula have equal length, *i.e.*, equal laser power. While apparently swinging in unison in the beginning, after a while the atoms get "out of sync." In the quantum picture, the energy $h\nu$ between the qubit states $|0\rangle$ and $|1\rangle$ is not the same for all qubits. While such optical lattices for atoms have been around for 25 years, the atomic swings themselves have never been successfully used to store quantum information for a significant time.

FIGURE 2:
Two pendula, coupled by a spring, feature two characteristic modes of motion: The pendula can swing relative to each other (the "vibrational mode," left), where the spring length oscillates, or they can swing together in unison, with the spring length fixed (the "center of mass mode," right). In quantum mechanics, corresponding modes exist, and are described by quantum mechanical waves (artistic rendition). These two quantum states of motion define a qubit. [Credit: Sampson Wilcox, MIT RLE]





The double pendulum

We need a new idea. What could be better than a single pendulum to keep time? Indeed, two pendula! If we attach a spring between two pendula (*Fig. 2*), there are two natural ways in which they swing as a couple. They can swing together in sync ("center of mass mode") and they can swing relative to each other ("vibrational mode"). For identical pendula, the difference in frequency between these two ways of swinging only depends on the strength of the spring and the pendulum mass.

But where can one find two absolutely identical pendula? Very simply, take two atoms and put them into the same laser trap. A wonderful fact about atoms is that they are completely indistinguishable. They have exactly the same mass, and, if placed in the same trap, they will feel exactly the same force. In our analogy, our two "atomic" pendula thus have the same string length.

We can now realize the above idea, and use the two different forms of coupled motion as our qubit. The idea is shown in Figure 3. One qubit state has each atom occupy state $|1\rangle$ of the trap (we could write this $|1\rangle|1\rangle$), the other qubit state has one atom in the ground state $|0\rangle$, the other in the second excited state $|2\rangle$ (we write $|0\rangle|2\rangle$). These two states have the same energy, so the overall frequency of their swing, and the value of the laser power, no longer matters. As we show in [2], this remains true if we include anharmonic corrections, whose presence Huygens first noted for the classical pendulum, and Heisenberg had worked out for the quantum pendulum [1] in 1925. The resulting energy difference only depends on Planck's constant h, the mass of the atoms, and the geometry of the laser trap. If in addition the atoms interact with each other, this acts like the "spring" of the classical example, favoring as qubit states the relative and the center of mass motion of the atom pair.

Indistinguishable atoms

One last challenge remains: How to fill an array of laser traps uniformly with two atoms per well? Not one (or none), not three, but two? At this point in the story three more physicist enter the scene: Wolfgang Pauli,

FIGURE 3:
The advantage of using pairs of atoms. Using the oscillator states of a single atom as qubit states is not a stable approach, since the frequency ν will differ slightly from one to the next qubit. However, using two atoms in each well gives access to two states, each having two units of vibration: $|0\rangle|2\rangle$, where one atom is in the ground, the other in the second excited state, and $|1\rangle|1\rangle$, where each atom is in the first excited state. The energy difference between the states is negligible on the scale of $h\nu$ and constant for each well. [Credit: Thomas Hartke, Zwierlein Group, MIT]

Enrico Fermi and Paul Dirac. Pauli's principle dictates that no two electrons can occupy one and the same quantum state. This principle underlies the periodic system of the elements. While the single electron in hydrogen has nothing to worry, the two electrons in helium must be in two different spin states ("up" and "down") to be able to occupy one and the same shell around the nucleus. The third electron in lithium sadly finds no more room there and must occupy the next higher shell. Fermi and Dirac independently applied this principle to motional states. As Pauli proved, his principle affects all particles with half-integer spin, which are called fermions. Examples are electrons, protons, neutrons and atoms containing an odd number of these building blocks, such as ^{3}He, ^{6}Li and ^{40}K.

In our recent experiment [2], we loaded a gas of fermionic ^{40}K atoms containing two spin states into an optical lattice. Using fermions ensured that at most two atoms, one of each spin state, would ever be found in a given lattice well—just like the two electrons in helium. Cooling the gas and tuning the density, we could fill large arrays of 400 atom pairs, all occupying the motional ground state $|0\rangle$ of the trap. In this way, the Pauli principle allowed for the high-fidelity initialization of our "fermion pair quantum register."

Controlling interactions

To "load the spring" between our double pendula, that is, to bring the atoms into relative vibration, we smoothly turn on repulsive interactions between them. This way we can initialize all atom pairs simultaneously in the qubit state $|0\rangle|2\rangle$. Controlling interatomic interactions exploits a "Feshbach resonance," a rather amazing tool named after Herman Feshbach, who was a renowned theoretical physicist at MIT. By simply applying a magnetic field in the lab, one brings a molecular state into resonance with the two colliding atoms. Resonant interactions are induced, as strong as quantum mechanics allows. Feshbach resonances, described alongside Frank Wilczek's article on Feshbach in the 2006 issue of *physics@mit*, allowed the creation of superfluids of atomic Fermi gases (see the 2006 and 2011 issues).

We are now ready to test our new "double pendulum qubits." That means we need to prepare a quantum superposition of the two ways of motion, relative and center of mass, and watch how long that superposition remains intact. The first to directly drive a system into a quantum superposition of two states was Isaac Rabi, who invented the nuclear magnetic resonance technique: an oscillating magnetic field drives nuclear spin transitions in atoms. In our double pendulum analogy, it is the spring that couples the two pendula—so modulating the spring will cause "Rabi transitions" and the pendula will alter their state of motion.

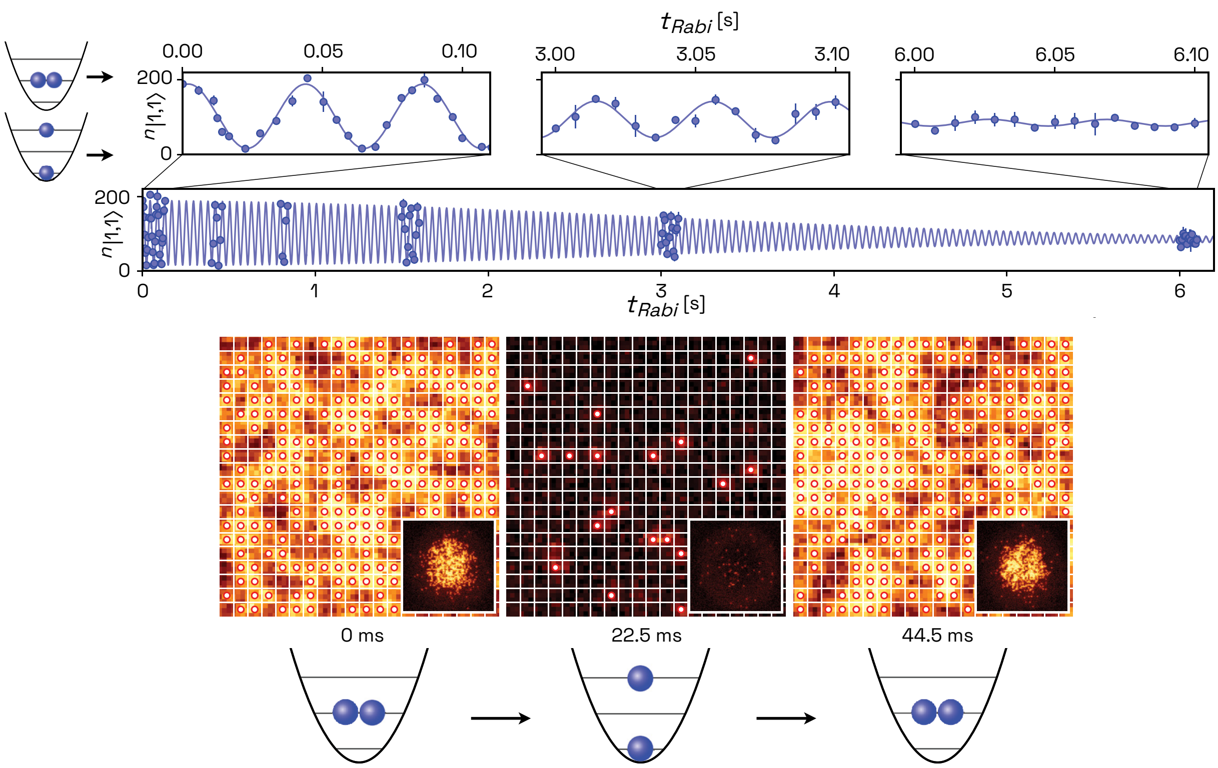
For the two atoms in the laser trap, this means that we need to modulate the interactions between atoms, their "spring force," and the Feshbach resonance mechanism allows just that. So as in Rabi's experiments it is a modulation of the magnetic field that drives a Rabi transition, only here it is transitions between two motional states of atom pairs, instead of nuclear spin transitions.

Performing this Rabi oscillation on our array of 400 atom pairs, we observed coherence times between the two motional states of the pairs on the order of ten seconds (*Fig. 4*). Every atom pair in the experiment was resolved under the microscope. The motional state was measured by converting relative and center-of-mass modes into bright and dark spots on the camera, respectively. This again made use of the Feshbach resonance, by converting the center-of-mass motion, where atoms are already close, into a tightly bound molecule, invisible to our fluorescence imaging.

Coherent vibrations

Whenever one has robust coherence between two quantum states, one can measure their energy difference in pristine fashion. This is the principle of atomic clocks, which employ Norman Ramsey's technique of "separated oscillatory fields." A first Rabi pulse creates a superposition state, which is allowed to freely evolve for some time, before a second Rabi pulse allows to read out the population in each qubit state. Here we used this technique to precisely measure the anharmonicity of the atom trap, which was indeed found insensitive to the laser power and given by the value predicted by Planck's constant, the atomic mass and the geometry of the trap. We were also able to precisely measure the energy of the weakly bound molecular state causing the Feshbach resonance. In this regime, our qubit was a coherent superposition of two free atoms and a tightly bound molecule. The coherence time was long enough to allow for 25 000 Ramsey oscillations.

FIGURE 4:
Rabi oscillations between motional states of atom pairs. By modulating the interactions between atoms (the "spring force" of the classical analogy), coherent superpositions of vibrational states are created that are seen to persist for many seconds. The top graph shows the recorded number of pairs in the |1⟩|1⟩ motional state. The three images below show snapshots of the quantum register at various times. The evolution of the quantum states from |1⟩|1⟩ to |0⟩|2⟩ and back is indicated.



Quantum computing with vibrating atoms

In the near future, we will develop methods to make two different atom pairs interact to realize two-qubit gates. Together with the demonstrated control over the atom pair qubit this would constitute an actual quantum computer. Furthermore, the method of using two particles instead of one to define a single logical qubit is going to be fruitful, even on other platforms. For example, superconducting qubits are also realized in an energy landscape that has the form of an anharmonic oscillator.

Using more deeply bound molecular states one can imagine arrays of precise molecular clocks. These could even run on various molecular transitions simultaneously, yielding parallel atom pair clocks "ticking" from kHz to hundreds of THz, enabling precision metrology largely shielded from laboratory noise.

Finally, increased control over atoms in optical lattices will enhance our ability to tackle paradigmatic problems in condensed matter and nuclear physics that cannot be solved on a classical computer. Fermions are particularly difficult to handle theoretically due to the Pauli principle and strong interactions. A famous example is the doped Hubbard model of mobile fermions hopping on a lattice and interacting when two unlike atoms meet on the same site. Despite the simplicity of the setting, the model has to this day not been solved in general. However, it is believed to hold the key to understanding high-temperature superconductivity. The quantum register described above is in fact built on top of a Fermi-Hubbard quantum simulator [3], which allows exploring the equation of state [4], correlations [5], [6] and transport properties [7] of strongly interacting fermions. Maybe the ability to create coherent motional states of fermion pairs will enable new insights also into the origins of superconductivity at strong coupling.

Looking back, we have come a long way since Galileo has watched that chandelier swing in Pisa. But still to this day, the physics of the pendulum amazes and inspires us in our endeavor to understand nature.

Collaborators with Prof. Martin Zwierlein on the quantum register project are MIT physics graduate students Thomas Hartke, Botond Oreg and Carter Turnbaugh, and postdoctoral associate Ningyuan Jia.

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MARTIN ZWIERLEIN studied physics in Bonn and at ENS-Paris, and received his PhD in experimental atomic physics from MIT in 2007, with a thesis supervised by Wolfgang Ketterle on the observation of superfluidity in atomic Fermi gases. He joined the MIT physics department in 2007 (tenure 2012, Full Professor 2013). Since 2018 he holds the Thomas A. Frank (1977) Chair of Physics. His research interests lie in strongly interacting quantum gases of atoms and molecules, *e.g.*, unitary Fermi gases, Bose-Fermi mixtures, Hubbard quantum simulators and quantum Hall physics with neutral atoms. Zwierlein's awards include the I. I. Rabi Prize of the American Physical Society (2017), the Vannevar Bush Faculty Fellowship (2019), and the Alexander von Humboldt Research Prize (2020).

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