GNUPAT Oral Exam Guidelines[1]

The GNUPAT oral exam serves the purpose of assessing a student's readiness to advance to full-time research. There are two aspects to this readiness. First, the examiners will test the depth of the student's knowledge of the GNUPAT field. Second, the examiners aim to test the ability of the student to reason effectively in real time from simpler to more difficult topics. In preparing for this exam, students will learn to discuss theoretical physics and teach it to their peers, and gain experience in scientific presentation.

The list of topics at the end of this document represents the GNUPAT faculty's collective view of what the core knowledge that all GNUPAT students should possess is; it emphasizes a foundational understanding rather than detailed minutiae. Students should be prepared to reason from this core.

The chair of the examination committee will meet with the cohort of students taking the exam at the beginning of the semester and students may consult as needed with the chair for clarification about topics and procedures. Students may find the 8.323–5 and 8.962 classes useful in preparation (to a lesser extent, 8.701/8.811 may provide interesting background material of a more phenomenological nature but are not necessary). The members of the examination committee can also provide suggestions for additional study resources. Students are also encouraged to discuss the exam with their research advisor.

Format of the exam

The exam consists of a presentation of a pre-assigned topic and a selection of questions from the committee on the topics below. The exam is approximately two hours in duration and no notes or other materials are permitted. The committee consists of three faculty (the student's research advisor can be present in the room during the examination but cannot speak or vote).

1) The student is given a topic for presentation at least two weeks prior to the scheduled exam date. The topic is chosen by the committee, and will be grounded in the topic list below but may go deeper or explore related ideas. Committee members can suggest additional references as needed once the topic is given. The student is expected to prepare a short blackboard presentation on the topic that, given without interruptions, should last no longer than 15 minutes. The exam will begin with this presentation. The committee will ask questions, mostly related to the exposition, throughout.

2) After the presentation part of the exam is concluded, the committee will proceed to the main part, with each of the three committee members asking a series of questions for approximately thirty minutes on the topics listed below. Students should anticipate a fairly evenly distributed coverage of the three areas in the topic list but should bear in mind that many good questions span two or more areas.

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[1] These guidelines apply to students entering MIT in 2023 and beyond.

During the exam, students should not be afraid to admit what they do not know and are encouraged to engage the committee for clarification (the exam is not a test of English proficiency!) and show their thought process. Long computations are never part of the exam.

Upon conclusion of the examination, the student will be asked to leave the room. The committee will deliberate and decide on a grade (Pass or Fail). Deliberations involve assigning grades (A,B,C,F) to the presentation and to each of the three sets of questions. To pass the exam, students must have at most one C, and no Fs. The student will be told their grade by the end of day or sooner. The committee chair will provide written feedback on the performance regardless of the outcome and be available to discuss the exam with the student.

The oral exam is taken in a student's second or third year at MIT. Two attempts are allowed and the first attempt must be taken by the end of the first term of the third year. If a student does not pass the exam on their first attempt, a second attempt must be made by the following semester (if the first attempt was taken in the first term of the second year, or earlier, the student may postpone the second attempt until the beginning of the third year).

Topic List

The level of knowledge of the topics in the list below is expected to be the average of the level at which that topic is presented in the following textbooks:

1) QFT & SM: Peskin & Schroeder, Schwartz, Srednicki, Weinberg (volumes 1&2)
2) GR: Carroll, Wald

A) Fundamentals of QFT

- Free and interacting Lagrangians involving real/complex scalars, Dirac/Majorana/Weyl fermions, abelian/non-abelian gauge fields.

- Canonical and path integral quantizations of Lagrangian field theories.

- Correlation functions, partition functions, scattering amplitudes, cross sections, and decay rates. LSZ reduction.

- Perturbation theory and Feynman rules. 1PI effective action. Perturbative unitary (optical theorem). Gauge fixing via the Fadeev-Popov procedure

- Global symmetry in field theory: Poincare invariance, spin/helicity (little groups), Noether's theorem, Ward identities (constraints on correlation functions involving a conserved current), background gauge fields, Goldstone's theorem, discrete spacetime symmetries (P, T, C). Basics of Lie Groups, Lie Algebras, representations.

- Renormalization group: UV divergences, renormalization (counterterms, schemes, etc.), beta functions, conformal symmetry and scaling at fixed points, Wilsonian renormalization group, renormalization group equations, asymptotic freedom.

- Basics of effective field theory. Identifying leading terms in an effective action given some IR degrees of freedom obeying certain symmetries, matching between UV and IR. Simple examples such as Euler-Heisenberg, Fermi theory.

- Phases of gauge theories: Coulomb (massless gauge bosons, power-law decay of potential between charges), confined (area law, flux tubes, absence of charged states, possible deconfinement at high temperature), Higgsed (unitary gauge, low-energy spectrum in the Higgs vacuum).

- Topology of gauge theory: magnetic monopoles, instantons, and theta angles.

B) Standard Model

- The gauge structure and particle content of the SM (how all fundamental fields and low-energy particles transform under the SU(3)xSU(2)xU(1) gauge group).

- Lagrangian and Feynman rules for SM interactions. Drawing diagrams associated with tree and loop level processes. Estimating rates for simple tree-level processes such as n → p e- nubar and e+e- → hadrons. Electron/muon g-2.

- Global symmetries of the standard model: C, P, T, B, L, isospin (including which combinations of these are exact and which are approximate).

- Higgs mechanism in the SM. Mass generation for gauge bosons and fermions, Higgs boson mass and interactions. CKM matrix (not numerical values). Rough values of masses of fundamental particles.

- Anomalies: chiral anomaly and neutral pion decay, explanation of the eta' mass via instantons, necessity of canceling gauge anomalies.

- Perturbative QCD: asymptotic freedom, IR divergences . Basics of parton distribution functions, jets, operator product expansion, and factorization.

- Light hadron spectrum: proton, neutrons, pions, SU(2) isospin in the limit of m_u=m_d=0 and m_s=∞.

- Chiral symmetry breaking and the pion lagrangian, including the lowest-dimension pion interactions.

- Qualitative knowledge of experimental evidence for SM:

- Cosmic rays (antiparticles, hadrons),
  - R ratio (N_c=3, quarkonia resonances),
  - Beta decay (weak interactions are left handed),
  - DIS (point like partons),
  - LHC production of Higgs (existence of Higgs),
  - Solar neutrino oscillations (neutrino masses and mixings).

- Neutrinos: difference between Dirac and Majorana masses, connection to B-L, oscillation in vacuum. PMNS matrix.

C) General Relativity

- (Pseudo-)Riemannian geometry: metric, parallel transport, connection, curvature, geodesics.

- Energy-momentum tensor: symmetry, conservation, energy conditions, relation to the matter action.

- Einstein field equations and Einstein-Hilbert action.

- Weak field and Newtonian limits.

- Stationary black hole solutions:
  - Schwarzschild (including maximal extension via Kruskal-Szekeres coords),
  - Reissner-Nordstrom,
  - Qualitative features of Kerr (extremality bound, ergoregion, Penrose process).

- Maximally symmetric solutions: de Sitter and anti de Sitter.

- Basics of causal structure: conformal diagrams, causal definition of horizons, global hyperbolicity, statements of the area theorem and the Penrose singularity theorem.

- Deflection of light. Shift of perihelion in orbits.

- Gravitational waves: polarizations, quadrupole nature of sources, amplitude of the LIGO signal.

- Homogeneity and isotropy of the universe. FRW equations including matter, radiation, curvature, vacuum energy. Cosmological redshift and expansion history of the universe. Basics of inflation (flatness and horizon problems, slow-roll conditions in single-field inflation, idea of reheating but not specific mechanisms) Not included: density perturbations, origin of the cosmic microwave background, big bang nucleosynthesis.

Appendix A: Sample exam questions

Phases of gauge theories:

a) Consider a U(1) gauge theory. What is the Lagrangian? What are the propagating degrees of freedom?
b) Now add a complex scalar field of charge 1. What is the Lagrangian? What is the spectrum of the theory in the Coulomb phase? The Higgs phase? What is the meaning of these phases in terms of the force between charged particles?
c) Let's switch to a SU(2) gauge theory. What is the Lagrangian? What are the propagating degrees of freedom? What are the interactions?
d) Now add a complex scalar field in the fundamental representation of SU(2). What is the Lagrangian? What are the possible phases of this theory and what are the associated forces? What is the spectrum of the theory in the various phases? [Does your answer depend on the number of scalar fields?]
e) Let's work in the Higgs phase of this theory, which is a simplified version of the electroweak sector of the SM. What is the Lagrangian at scales below the mass of any Higgs bosons?
f) This Lagrangian (without the Higgs) is famously non-renormalizable. Experimentally, one way to see that this theory is pathological is that the WW -> WW scattering amplitude grows unbounded with energy. Draw the Feynman diagrams for WW -> WW scattering. Estimate this amplitude in the high energy limit.
g) Bonus: Your estimate above should give you g^2 (E/M)^4. The actual answer is g^2 (E/M)^2. But M = g v, so the answer is (E/v)^2, independent of the gauge coupling! Explain the physical reason for this.

Muon g-2:

a) What is the anomalous magnetic moment of a spin ½ particle?
b) What is the first QED contribution to electron g-2? Do you know its value?
c) Draw Feynman diagrams showing the leading contribution of each of the other SM particles to muon g-2 (W, Z, H, gluon, quark, neutrino). What are the relative importances of the W, H, and Z pieces?
d) Given the experimental measurement of muon g-2 is roughly four orders of magnitude less precise than that of electron g-2, why is the muon case more sensitive to BSM physics?
e) Concerning the diagrams you drew with quarks and gluons, what class of other diagrams are equally important and are diagrams the right way to think about these pieces?
f) How does the R-ratio (ie e+e- → hadrons) help determine this hadronic vacuum polarization contribution to g-2?

Geodesics in the Schwarzschild geometry

a) Explain why the norm of the tangent vector is conserved along a geodesic.

b) Explain why the inner product of the tangent vector with a Killing vector is also conserved along a geodesic.
c) Write down the Schwarzschild metric in 3+1 dimensions.
d) Write down the Killing vectors corresponding to time translations and rotations of the Schwarzschild metric.
e) What are the conserved quantities obtained by dotting these Killing vectors with a timelike geodesic parametrized by its proper time?
f) Using the conservation of the norm of the tangent vector, write down a first-order differential equation for the radius as a function of proper time. The conserved quantities from the previous part should appear in this differential equation.
g) Which terms in this equation vanish in the Newtonian limit? How do they modify the qualitative behavior of trajectories near a massive body in GR compared to Newtonian gravity?