MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics 8.01 ASE Equation Sheet

Kinematics:

$\vec{\mathbf{v}} = d\vec{\mathbf{R}}/dt,\ \vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$ Velocity, Acceleration

$v_x(t) - v_x(0) = \int_{t'=0}^{t'=t} a_x(t')\,dt'$ Velocity Change

$x(t) - x(0) = \int_{t'=0}^{t'=t} v_x(t')\,dt'$ Displacement

$\vec{\mathbf{a}} = \left[\frac{d^2r}{dt^2} - r(\frac{d\theta}{dt})^2\right]\hat{\mathbf{r}} + \left[r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right]\hat{\boldsymbol{\theta}}$

Acceleration in Polar Coordinates

Circular Motion Kinematics:

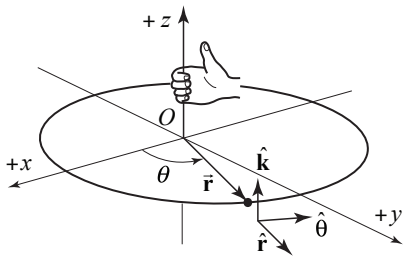
$\vec{\mathbf{R}} = r\,\hat{\mathbf{r}}$ Position

$\vec{\mathbf{v}} = r(d\theta/dt)\hat{\boldsymbol{\theta}} = v_\theta\hat{\boldsymbol{\theta}}$ Velocity

$\vec{\omega} = (d\theta/dt)\hat{\mathbf{k}} = \omega_z\hat{\mathbf{k}}$ Angular Velocity

$\omega = |\vec{\omega}| = |d\theta/dt|$ Angular Frequency

$\vec{\alpha} = (d\omega_z/dt)\hat{\mathbf{k}} = (d^2\theta/dt^2)\hat{\mathbf{k}} = \alpha_z\hat{\mathbf{k}}$ Angular Accel.



Force and Momentum:

$\vec{\mathbf{F}}_{\text{ext}} = m\,\vec{\mathbf{a}} = d\vec{\mathbf{p}}^{\,\text{sys}}/dt$ Newton's 2nd Law

$\vec{\mathbf{F}}_{1\,2} = -\vec{\mathbf{F}}_{2\,1}$ Newton's 3rd Law

$\vec{\mathbf{F}}_G = -(Gm_1\,m_2/r^2\,)\hat{\mathbf{r}}$ Gravitation

$\vec{\mathbf{F}}_s = -k\Delta x\,\hat{\mathbf{i}}$ Hooke's Law

$\vec{\mathbf{p}} = m\vec{\mathbf{v}},\qquad \vec{\mathbf{p}}^{\,\text{sys}} = \sum_{j=1}^{N} m_j\,\vec{\mathbf{v}}_j$ Momentum

$\vec{\mathbf{I}} = \vec{\mathbf{p}}(t_f) - \vec{\mathbf{p}}(t_i) = \int_{t=t_i}^{t=t_f} \vec{\mathbf{F}}^{\text{ext}}(t)dt$ Impulse

$\mathrm{P} = F/A$ Pressure

Energy:

$K = (1/2)\,mv^2$ Translational Kinetic

$W = \int_A^B \vec{\mathbf{F}}\cdot d\vec{\mathbf{s}} = \Delta K$ Work-Energy Principle

$\Delta U \equiv U(B) - U(A) \equiv -W_{\text{c}} = -\int_A^B \vec{\mathbf{F}}_c\cdot d\vec{\mathbf{s}}$ Potential

$U_g(y) = mgy \qquad U_g(0) = 0$ Uniform Gravity

$U_G(r) = -Gm_1m_2/r \qquad U_G(\infty) = 0$ Gravity

$U_s(x) = (1/2)\,kx^2 \qquad U_s(0) = 0$ Springs

$W_{\text{nc}} = \Delta K + \Delta U^{\text{total}} = \Delta E^{\text{mech}}$ Conservation

Rigid Body Motion About the z-axis:

$I_S = \int_{\text{body}} r^2 dm$ Moment of Inertia

$I_{sphere,cm} = \frac{2}{3}MR^2$ Sphere

$I_{cylinder,cm} = \frac{1}{2}MR^2$ Cylinder

$I_{hoop,cm} = MR^2$ Hoop

$I_{rod,cm} = \frac{1}{12}ML^2$ Rod, transverse through center

$I_S = md^2 + I_{cm}$ Parallel Axis Theorem

$\vec{\tau}_S = \vec{\mathbf{R}}_{S,F}\times\vec{\mathbf{F}}$ Torque about S

$\tau_{S,z} = I_S\alpha_z$ Rigid Body, Fixed Axis

$K_S^{\text{rot}} = (1/2)I_S\omega^2$ Rotational Kinetic Energy

Angular Momentum:

$\vec{\mathbf{L}}_S = \vec{\mathbf{R}}_S\times m\vec{\mathbf{v}}$ Point Mass about S

$\vec{\mathbf{L}}_{\text{axis}} = I_{\text{axis}}\,\vec{\omega} = I_z\omega_z\hat{\mathbf{k}}$ Rigid Body

$\vec{\tau}_S^{\,\text{ext}} = d\vec{\mathbf{L}}_S^{\,\text{sys}}/dt$ Torque Relation

Translation and Rotation:

$$\vec{\mathbf{R}}_{\rm cm} = \tfrac{1}{m^{\rm total}} \sum_{j=1}^{N} m_j \vec{\mathbf{R}}_j = \tfrac{1}{m^{\rm total}} \int_{\rm body} dm\, \vec{\mathbf{R}}$$

Center of Mass

$$\vec{\mathbf{v}}_{\rm cm} = \tfrac{1}{m^{\rm total}} \sum_{j=1}^{N} m_j\, \vec{\mathbf{v}}_j = \tfrac{1}{m^{\rm total}} \int_{\rm body} dm\, \vec{\mathbf{v}}$$

Center of Mass Velocity

$$K = (1/2)m{v_{cm}}^2 + (1/2)I_{cm}\omega^2$$ Kinetic Energy

$$\vec{\mathbf{L}}_S = \vec{\mathbf{R}}_{S,cm} \times m\vec{\mathbf{v}}_{cm} + I_{cm}\vec{\omega}$$ Angular Momentum

Simple Harmonic Oscillator:

$$d^2x/dt^2 + {\omega_0}^2 x = 0$$ Equation of Motion

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$ Solution

$$\omega_0 = \sqrt{\tfrac{k}{m}}$$ Spring Frequency

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$ Period and Frequency Relations

Taylor Series:

$$f(x) \approx f(a) + \left.\frac{df}{dx}\right|_{x=a}(x-a) + \tfrac{1}{2!}\left.\frac{d^2 f}{dx^2}\right|_{x=a}(x-a)^2$$

Kepler's Laws:

$$\tfrac{dA}{dt} = \tfrac{L_z}{2\mu}$$ Equal Area

$$T^2 = \tfrac{4\pi^2 a^3}{G(m_1+m_2)}$$ Period

Rotating Reference Frames:

$$\vec{\mathbf{v}}_{\rm inertial} = \vec{\mathbf{v}}'_{\rm rotating} + (\vec{\omega} \times \vec{\mathbf{r}})$$ Transformation

$$\vec{\mathbf{F}}_{\rm centrifugal} = -m(\vec{\omega} \times (\vec{\omega} \times \vec{\mathbf{r}}))$$ Centrifugal Force

$$\vec{\mathbf{F}}_{\rm coriolis} = -2m\vec{\omega} \times \vec{\mathbf{v}}'$$ Coriolis Force

$$\vec{\mathbf{F}}_{\rm phy} - m(\vec{\omega} \times (\vec{\omega} \times \vec{\mathbf{r}})) - 2m\vec{\omega} \times \vec{\mathbf{v}}' = m\vec{\mathbf{a}}'$$

Newton's 2nd Law